Production and financial policies under asymmetric information

Abstract  We propose an extension of the standard general equilibrium model with production and incomplete markets to situations in which (i) private investors have limited information on the returns of specific assets, (ii) managers of firms have limited information on the preferences of individual shareholders. The extension is obtained by the assumption that firms are not traded directly but grouped into ‘sectorial’ funds. In our model the financial policy of the firm is not irrelevant. We establish the existence of equilibria and discuss the nature of the inefficiencies introduced by the presence of asymmetric information. We also illustrate the properties of the model in three simple examples.

Keywords  Decision-making under uncertainty · Asymmetric and private information · Incomplete markets · Equilibrium

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Introduction

We introduce information asymmetries in the standard general equilibrium model with production and incomplete markets (see e.g., Geanakoplos et al. 1990). In that model, shares of stock of individual firms are traded on a competitive stock exchange; whereas the firms choose production plans that are Pareto-efficient from the viewpoint of owners—shareholders.

Two types of information failures naturally come to mind in this framework: (i) private investors may have limited information about the returns from specific assets; they cannot differentiate finely individual firms belonging to a broad class, like an industrial sector; (ii) managers of firms may have limited information about the identity and preferences of individual investors—shareholders; they do not know whether and how the preferences of anonymous shareholders may differ from those of identified shareholders.

In this paper, we introduce the simplest possible framework encompassing these features. Some assets (firms) are not traded, but are grouped into funds traded on a stock exchange. The allocation of individual assets to the funds is an a priori given partition. Elements of the partition are labeled “sectors”. Each firm is initially owned by a single individual. The firm can raise outside funding by selling equity to the fund in which it belongs. In order to model limited information on the part of investors, we assume that the equity of all firms in a given sector is bought by the fund at a single price. That price emerges from the market value of the fund on the stock exchange. Given that price, the initial owners of a firm choose the share of their equity supplied to the fund. As for the funds, they act passively, applying to their equity purchases the price emerging from their market value, and refraining from interfering with the decisions of individual firms. Finally, the individuals trade in shares of the funds and a safe bond.

In interpreting this simple model, one may think about the funds as veils reflecting the inability of investors to differentiate firms belonging to the same sector on the one hand, the inability of firms to identify anonymous shareholders on the other hand. The resulting inefficiencies are then due to these information asymmetries.

This simple structure is based on the work of Dubey, Geanakoplos and Shubik in the nineties (see Dubey et al. (2005)), then Bisin and Gottardi (1999) and Bisin et al. (2001), on equilibrium in pure exchange economies with asymmetric information. As will be seen, the extension to production economies enriches the analysis substantively. Also, the financial decision (how much equity is supplied to the fund) is of genuine interest.

Building on the standard general equilibrium model with production and incomplete markets, we first adapt the model to the specification just outlined (Sect. 1). An equilibrium still requires that asset markets clear, and firm decisions are optimal for the initial owner. Under standard assumptions, equilibria exist.

We then discuss the definition of “constrained Pareto efficiency” (Sect. 2) by comparing the first-order necessary conditions for constrained Pareto optimality with the corresponding conditions for equilibrium of firms and investors. Not surprisingly, they differ: the former take into account the implications of firm-level decisions for the anonymous shareholders holding fund shares, the latter do not. On
this observation we provide a short argument for generic constrained suboptimality of competitive equilibria. Then, in order to bring out the specific inefficiencies resulting from information asymmetries, we analyze two special cases where equilibria of production and asset exchange are constrained-Pareto efficient, namely: (i) the model of multiplicative uncertainty (a special case of “partial spanning”) in Diamond (1967); (ii) the model of mean-variance investor preferences and non-stochastic endowments known as CAPM. For both cases, we show by means of simple examples that the efficiency result does not carry over to our model. In case (i), this is due to inefficient investment levels; in case (ii), it is due to inefficient supply of equity. For completeness, we also illustrate a third case in which inefficiencies result from the state distribution of the returns from investments.

Taking into account the preferences of all final shareholders (direct or indirect) corresponds to the decision criterion for firms introduced by Drèze (1974). Pareto efficiency for all final shareholders, as postulated under that criterion, would require the funds to elicit the preferences (shadow prices) of fund holders, then to intervene as active shareholders of the firms on the basis of these preferences (duly aggregated). We establish that such intervention would eliminate the additional inefficiencies due to information asymmetries but of course not those already present in the standard model (Sect. 3). This closes the comparison between efficiency under our specification and under the standard specification.

1 The economy and equilibrium

The economy has two time periods. Uncertainty is over \( S \) possible states of nature, \( S = \{1, \ldots, s, \ldots, S\} \), and is resolved in the second period. There are \( J \) firms \( J = \{1, \ldots, j, \ldots, J\} \), and \( H \) consumers, \( H = \{1, \ldots, h, \ldots, H\} \), with \( J \leq H \). There is one commodity per state.

A firm \( j \in J \) is characterized by a closed and convex production set \( Y^j \subset \mathbb{R}^{S+1} \) with the properties that \( 0 \in Y^j \) and \( Y^j \cap \mathbb{R}_{++}^{S+1} = \{0\} \). A feasible production plan is \( y = (−y_0, y_1) \), where \( y_0 \in \mathbb{R}_+ \) denotes input and \( y_1 \in \mathbb{R}_+^S \) a state contingent output vector. We assume that production possibilities can equivalently be represented by a non-decreasing, quasi-convex, differentiable transformation function, \( F^j : \mathbb{R}^{S+1} \rightarrow \mathbb{R} \) with \( F^j(0) = 0 \). Each consumer \( h \in H \) is fully characterized by \( (u^h, \omega^h, s^h) \). Here \( u^h : \mathbb{R}_+^{S+1} \rightarrow \mathbb{R} \) is a monotonically increasing, quasi-concave and differentiable utility function. Commodities endowment is \( \omega^h \in \mathbb{R}_+^{S+1} \). We make the simplifying assumption that each firm is initially owned by a single individual, and that each individual owns at most one firm.\(^2\) Therefore, denoting by \( h \in \{1, \ldots, J\} \) the firm owned by individual \( h \), ownership shares are \( s^h = (\ldots, s^h_j, \ldots) \in \{0, 1\}^J \), with \( s^h_h = 1 \), and \( s^h_j = 0 \) for all \( j \) different from \( h \).

1.1 Symmetric information

In the standard model of equilibrium with incomplete markets (GEI), information is symmetric. Firms are traded on the stock market and each investor knows the distribution of profits across the possible states of nature.

\(^2\) The extension of our model to the case of multiple owners can be found in our working paper, Drèze et al. (2004).
Given a technology $Y^j$, the owner of firm $j$ optimally chooses production and financing jointly with his consumption and asset portfolio. Investment costs, $y_0^j$, are financed by direct contributions of the initial owner and by outside financing. The only form of outside financing is represented by the possibility of the firm to sell equities, claims on future returns from production. A share $\zeta^j$ of equities issued today, at price $q_j$, entitles the firm to receive $\left(q_j + y_0^j\right)\zeta^j$. Investors are entitled to a share of future returns $\zeta^j y_{1+s}^j$.

At given stock prices $q \in \mathbb{R}_+^J$, consumer $h$ chooses a consumption plan $x^h \in \mathbb{R}_+^{S+1}$, a portfolio of stocks $\theta^h \in \mathbb{R}_+^J$, a production plan $y^h \in Y^h$ and a financial policy for his firm $\zeta^h \in [0, 1]$ to solve

$$\max_{x^h, \theta^h, y^h, \zeta^h} u^h(x^h) \text{s.t.}$$

$$x_0^h - o_0^h - \Pi_0^h + \sum_j \left(q_j^j + y_0^j\right)\theta_j^h = 0$$

$$x_s^h - o_s^h - \Pi_s^h - \sum_j y_{1+s}^j \theta_j^h = 0, \quad s \in S$$

where

$$\Pi_0^h = \left[-\left(1 - \zeta^h\right) y_0^h + q^h \zeta^h\right]$$

$$\Pi_1^h = \left(1 - \zeta^h\right) y_1^h$$

denote firm $h$’s retained earnings.

Notice that when choosing $\theta_h^h > 0$, the initial owner participates in the outside financing of his own firm, at the same terms as any other investor. From the budget constraint, one sees that what really matters is the ‘net’ outside financing, $\left(\zeta^h - \theta_h^h\right)$. In particular, we can without loss of generality, assume that all shares are sold, $\zeta^h = 1$, and that individual $h$ buys back $\theta_h^h$ of them. With this convention we obtain:

$$\Pi_0^h = q^h$$

$$\Pi_1^h = 0$$

and we recover the standard form of the budget constraint, in which the financial policy of the firm does not appear explicitly. This is an instance of the Modigliani–Miller principle on the irrelevance of the firm’s financial policy when firms and individuals have access to the same set of financial instruments.\(^3\)

1.2 Asymmetric information

We want to extend the model to capture the asymmetric information between initial owners and potential investors. Initial owners have inside information on their firm production plans which they may not be able to credibly communicate to the market.

\(^3\) See e.g., the discussion in Magill and Shafer (1991).
We propose to model this by assuming that firms are partitioned in \( M \) types, or sectors, \( \mathcal{M} = \{1, \ldots, m, \ldots, M\} \). Because of asymmetric information, investors are only willing to pay a (sector) price \( q_m^m \) for a promise to deliver \( y_j^j \), for all \( j \in m \). Thus, a firm in sector \( m \) can only obtain outside financing by issuing equities at the sector price \( q_m^m \). The economy is defined by

\[
\mathcal{E} = \left\{ \mathcal{H}, \mathcal{S}, \mathcal{M}, \mathcal{J}, (u^h, \omega^h, s^h), (Y^j) \right\}.
\]

Markets

In the first period agents trade in assets. There are \( M \) security markets where financial resources are intermediated between consumers and firms. Each security is traded competitively at price \( q_m^m \) and yields a state dependent payoff \( r_m^m \in \mathbb{R}^{S+1} \), which reflects the average return from investment in sector \( m \). We shall later specify \( r_m^m \) as the outcome of a financial intermediation activity.

To simplify computations in the examples, we assume that there is also a market for a riskless bond, \( m = 0 \), with returns \( r_0^0 = 0, r_s^0 = 1 \) for all \( s \in S \).

The asset structure is described by the \((S + 1) \times (M + 1)\) matrix of securities’ returns \( R = [r^0, \ldots, r^m, \ldots] \). Markets are incomplete, \( M + 1 < S \).

Consumption and production choices

As in the full information benchmark, investment costs, \( y_j^j \), are financed by direct contributions of the initial owner and by outside financing. The important difference is that now the only form of outside financing for \( j \in m \) is to sell equities to the sectorial fund, at a price \( q_m^m \). A share \( \zeta_j \) of equities entitles the firm to receive \((q_m^m + y_j^j)\zeta_j \). Investors receive a share of future returns \( \zeta_j y_j^j \). We assume that control rights fully remain in the hands of initial owners.

Consumer \( h \) solves the following problem

\[
\max_{x^h, \theta^h, \zeta, y^h} \left( x^h \right) \text{s.t.}
\]

\[
(\lambda_0^h) \hspace{1cm} x_0^h - \omega_0^h - \Pi_0^h + \sum_{m \in \mathcal{M}} (q_m^m - r_0^m) \theta_m^h + q_0^h \theta_0^h = 0
\]

\[
(\lambda_s^h) \hspace{1cm} x_s^h - \omega_s^h - \Pi_s^h - \sum_{m \in \mathcal{M}} r_s^m \theta_m^h - \theta_0^h = 0, \quad s \in S
\]

\[
x^h \geq 0, \quad \theta_m^h \geq 0
\]

\[
y_j^j \in Y^j, \quad \zeta_j \in [0, 1]
\]

where \((\lambda_0^h, \ldots, \lambda_s^h, \ldots) \in \mathbb{R}^{S+1}_+\) are Lagrange multipliers, and the following definitions are used

\[
\Pi_0^h = \left[ -(1 - \zeta^h) y_0^h + q_m^m \zeta^h \right]
\]

\[
\Pi_1^h = (1 - \zeta^h) y_1^h.
\]
In our model initial shareholders are assumed to have perfect knowledge of the firm technology and choices. However, because initial shareholders’ information cannot be credibly conveyed to other potential investors, initial ownership is not directly tradable: as investor in the asset market, every agent faces the same set of sectorial pooled assets. Notice that, although firm shares are not traded directly, firm ownership provides an extra degree of risk sharing with respect to the one attainable on financial markets. This is limited by our assumption of no short sales of assets: trading in asset \( m \), the initial owner of firm \( j \) in sector \( m \) can only increase his final stake in firm \( j \) profits.

More importantly, the Modigliani–Miller principle does not hold in our setting. If firm \( h \) was the only one in sector \( m \) we would have \( q^m = q^j, r_0^m = -y_0^j, r_s^m = y_s^j, \theta^h = \theta^h \), and as before we would be able to simplify the analysis by setting \( \zeta^h = 1 \). On the other hand, if, as is to be expected in general, more than one firm is financed by the sectorial fund, the financial policy of the firm is not irrelevant. Individual \( h \) cannot undo the effect of any change in \( \zeta^h \) by a change in his portfolio \( \theta^h \). Only the aggregate sectorial assets are on the market, and we should not expect that the profit stream of a given firm can be exactly replicated: the two components of individual \( h \)’s financial policy are not perfect substitutes any more.

Financial intermediaries

Intermediaries are passive funds. An intermediary \( m \) buys claims on future returns from production activities in sector \( m \), at an initial outlay \( \sum_{j \in m} \xi^j (q^m + y_0^j) \). The intermediary raises funds through the asset market. In a notation consistent with the one used for firms, we model the intermediary as issuing \( \bar{\zeta}^m \equiv \sum_{j \in m} \xi^j \) “shares” at a unit price \( \tilde{q}^m \), with unitary returns:\(^4\)

\[ r^m = \sum_{j \in m} \frac{\xi^j}{\tilde{q}^m} y^j. \]

If consumer \( h \) buys \( \theta^h \) shares, market clearing requires \( \bar{\zeta}^m = \bar{\theta}^m \equiv \sum_{h} \theta^h \). The price \( \tilde{q}^m \) should see to that.

Note that market clearing for shares implies zero profit (i.e., no dividends) in the second period:

\[ \bar{\theta}^m r_1^m = \sum_{j \in m} \xi^j y^j. \quad (1) \]

In the first period, the accounting profit of fund \( m \) is:

\[ \tilde{q}^m \bar{\theta}^m - r_0^m \bar{\theta}^m - q^m \bar{\zeta}^m - \sum_{j \in m} \xi^j y_0^j = \Pi_0^m. \]

Here market clearing implies \( \tilde{q}^m - \frac{\Pi_0^m}{\bar{\theta}^m} = q^m \), that is the market price of a share of the fund net of dividend is always equal to the buying price of firms’ shares in the sector. It simplifies notation to use throughout this price net of dividends, \( q^m \), and omit any reference to the first period profits of the fund.

\(^4\) Remember that \( y^j = (-y_0^j, y_1^j) \), \( y_0^j \in \mathbb{R}_+ \), so that \( r_0^m \leq 0 \).
Equilibrium

We now give a definition of competitive equilibrium for our economy.

**Definition 1** A competitive equilibrium for an economy $\mathcal{E}$, is a profile of allocations, security prices, and returns $((x, \theta, y, \zeta), q, R)$ such that

1. $(x^h, \theta^h, \zeta, y)$ solves the individual problem at $(q, R)$, $h \in \mathcal{H}$,
2. funds’ returns are $r^m = \sum_{j \in m} \frac{\zeta^j}{\bar{\zeta}^m} y^j$ if $\bar{\zeta}^m > 0$, $m \in \mathcal{M}$,
3. asset markets clear, $\bar{\theta}^m = \bar{\zeta}^m$, $m \in \mathcal{M}$.

It is easy to verify that individual budget balance and asset market clearing imply commodity market clearing in every state.

**Assumption 2**

1. $\forall h, u^h : \mathbb{R}_{+}^{S+1} \to \mathbb{R}$ is continuous, quasi-concave, strictly monotone;
2. $\forall h, \omega^h \gg 0$;
3. $\forall j, Y^j$ is a closed, convex subset of $\mathbb{R}_{+}^{S+1}$ and $0 \in Y^j$;
4. $(\sum_h \omega^h + \sum_j Y^j) \cap \mathbb{R}_{+}^{S+1}$ is compact.

**Theorem 3** Under Assumption 2, for every economy $\mathcal{E}$ a competitive equilibrium exists.

The proof follows the one of Theorem 1 in Dubey et al. (2005). As in that paper, an equilibrium without fund asset trading can always be sustained by setting $q^m = 0$, $r^m = 0$ for all $m \in \mathcal{M}$, $\zeta^j = 0$ for all $j \in \mathcal{J}$. To prove existence of non-trivial equilibria, one has to deal with the discontinuity of the asset payoff when the denominator is zero. The complete argument can be found in our working paper.

2 Efficiency

In this section we introduce a notion of constrained efficiency that extends that of Diamond (1967) to our model.

Diamond was the first to study the welfare properties of equilibria in a production economy under incomplete markets. In his paper he defines an allocation as feasible if it can be attained by redistributing shares of the existing firms, operating at suitable levels. The resulting notion of constrained efficiency has become standard in the general equilibrium literature. Since Stiglitz (1982) it has been understood that, with multiple goods, incomplete markets equilibria may be constrained inefficient due to the existence of pecuniary externalities. A planner may attain a Pareto improvement by inducing a change in relative spot prices.\(^5\)

\(^5\) The argument for generic inefficiency has been developed by Geanakoplos and Polemarchakis (1986) for exchange economies and by Geanakoplos et al. (1990) for economies with production.
In our paper there is one commodity per state, and no role for an intervention through spot prices. Yet production and financial decisions do have external effects: a change in $(\zeta, y)$ affects the payoff of the sectorial fund and the planner will take into account these externalities. This is formalized in the following definitions.

**Definition 4** (Constrained feasibility) An allocation $(x, y, \theta, \zeta)$ is constrained feasible if it satisfies the following constraints:

\[
(\hat{\lambda}_0) \sum_h (x_h^0 - \omega_h^0 - r_0 \theta^h) + \sum_j (1 - \zeta^j) y_j^0 = 0
\]
\[
(q^m) \sum_h \theta^h_m - \sum_{j \in m} \zeta^j = 0, \quad m \in \mathcal{M}
\]
\[
(\hat{\theta}^0) \sum_h \theta^h_0 = 0
\]
\[
(\hat{\lambda}^h) x^h_s - \omega_h^s - r_s \theta^h - \theta^h_0 - (1 - \zeta^h) y^h_s = 0, \quad s \in \mathcal{S}, \ h \in \mathcal{H}
\]
\[
(\hat{v}^j) F^j(y^j) \leq 0, \quad j \in \mathcal{J}
\]
\[
r^m = \left(1 / \sum_{j \in m} \zeta^j \right) \sum_{j \in m} \zeta^j y^j, \quad y^j \in Y^j, \ m \in \mathcal{M}.
\]

**Definition 5** (Constrained Pareto Optimality) A constrained feasible allocation $(x, y, \theta, \zeta)$ is constrained Pareto optimal (CPO) if there does not exist a constrained feasible allocation $(x', y', \theta', \zeta')$ that Pareto dominates it.

The informational restrictions are taken into account by the fact that we do not allow the planner to create markets for the shares of individual firms. As for private agents, they are forced to use the existing sectorial funds.

By comparing the first order conditions at a constrained optimal solution and at an equilibrium of our model we identify a new potential source of suboptimality, specific to the asymmetric information setting. A CPO allocation satisfies the first order conditions of the problem of maximizing $\sum \alpha_j u_j$ over the set of constrained feasible allocations, for some vector $\alpha$ of welfare weights. At an interior solution of this problem, first order conditions are

\[
\theta^j_m: \hat{q}^m = \sum_{s \geq 0} \hat{\lambda}^j_s r^m_s, \quad j \in \mathcal{H}, \ m \in \{0\} \cup \mathcal{M}
\]
\[
x^j: \alpha^j \frac{\partial u^j}{\partial x^j}(x^j) = \hat{\lambda}^j, \quad j \in \mathcal{H}
\]
\[
y^j: \hat{v}^j \partial_x F^j = (1 - \zeta^j) \hat{\lambda}^j_s + \sum_h \theta^h_m \hat{\lambda}^j_s \frac{\partial r^m_s}{\partial y^j_s}, \quad s \in \{0\} \cup \mathcal{S}
\]
\[
\zeta^j: \hat{q}^m - \sum_{s \geq 0} \hat{\lambda}^j_s y^j_s + \sum_h \sum_{s \geq 0} \theta^h_m \hat{\lambda}^j_s \frac{\partial r^m_s}{\partial y^j_s} = 0, \quad j \in \mathcal{J}, \ m \in \mathcal{M}
\]

where $Du^j = (\partial_0 u^j, \ldots, \partial_{S} u^j, \ldots) \in \mathbb{R}^{S+1}$ is the vector of marginal utilities of $u^j(x^j)$, and where $y^j$ satisfies technological feasibility, $F^j(y^j) \leq 0$.

At an interior equilibrium, the following first order conditions for individual $h$ are satisfied:

\[
\theta^h_m: \gamma^h_0 q^m = \sum_{s \geq 0} \hat{\lambda}^h_s r^m_s, \quad m \in \{0\} \cup \mathcal{M}
\]
\[
x^h: Du^h(x^h) = \lambda^h
\]
and, for firm $j$:

$$y^j_s : v^j_s \partial F^j_s = (1 - \zeta^j) \lambda^j_s, \quad s \in \{0\} \cup S$$

$$\zeta^j : \lambda^j_0 q^m - \sum_{s \geq 0} \lambda^j_s y^j_s = 0 \quad (4)$$

Next, let Lagrange multipliers and asset prices satisfy $\lambda^j = (\hat{\lambda}_0/\alpha^j, \ldots, \hat{\lambda}_s/\alpha^j, \ldots) \in \mathbb{R}^{S+1}_+, q^m = \tilde{q}^m/\hat{\lambda}_0$, $v^j = \tilde{v}^j/\alpha^j$, for all $j$ and all $m$. Evaluating the equilibrium first order conditions at $(x, \theta, y, q, \lambda, v)$ and comparing them with the corresponding first order conditions in (2), one sees that they are the same up to some terms which are absent from (4). The absence of these terms represents a further source of inefficiency with respect to the well known ones which arise in standard economies with incomplete markets (see Drèze 1974; Geanakoplos et al. 1990).

Looking more closely, we see that the additional inefficiency emerges because the firm fails to take into account the effect of its investment and of its financial policy on the returns of the fund. Again, this may be interpreted either by saying that the firm simply acts competitively on the markets, taking the return and price from investing in the fund as given; or by considering that the firm lacks specific information on actual fund holders\footnote{This lack of information persists also if the firm is the only one intermediated through the fund. This implies that in our model the firm is not using the Drèze’s criterion, see our discussion in Sect. 3.} [namely the terms $\theta^j$’s in the first order conditions with respect to $\zeta$ and $y$ in (2)]. These sources of inefficiencies will be used to establish generic constrained suboptimality of competitive equilibria in the next theorem, a result not established for Drèze or Grossman–Hart equilibria in a one-good economy. Before stating the theorem, we need to introduce a technical assumption:\footnote{The existence of the set $\Omega$ can be proved along the lines of the proof of Theorem 2 in Geanakoplos et al. (1990). Indeed, the only additional step is to prove that $\zeta$ is interior, but this can be done reiterating the argument used to show that $\theta$ is interior. We therefore skip technical details.}

**Assumption 6** There exists a generic set of endowments, $\Omega$, such that for every $\omega$ in $\Omega$ equilibria are interior, meaning that

1. the payoff matrix is of full rank;
2. funds portfolios are strictly positive, $\theta \in \mathbb{R}^{HM}_{++}$, and firms financing decisions are interior, $\zeta \in (0, 1)^J$;
3. equilibrium variables locally behave as smooth functions of $\omega$.

**Theorem 7** (Generic constrained suboptimality) Under Assumption 6, there exists a generic set of economies parametrized by endowments such that for every one of such economies competitive equilibria are not constrained Pareto optimal.

**Proof** It suffices to show that there exists a generic set $\Omega^* \subset \Omega$ such that for every $\omega$ in $\Omega^*$, allocations and prices which satisfy first order conditions for an equilibrium do not typically satisfy first order conditions for optimality. To this end we claim that for every $\omega$ in $\Omega^*$, $\sum_h \theta^h_m \frac{\lambda^j_k}{\lambda^0} \frac{\partial r^m}{\partial y^j_s} \neq 0$ for all $m, j, s$. For suppose not, assume that
the latter be zero for some \( j \) and \( m \). Observe that 
\[
\sum_h \theta_m^h \lambda^h \sum \frac{\partial r_m^h}{\partial y_j} = \sum \frac{\partial r_m^h}{\partial y_j},
\]
and consider the following local perturbation of the endowments, \( d\omega : d\omega^h = Wd\theta^h \),
\( W \in \mathbb{R}^{(S+1) \times M} \), \( W_m^0 = q_m - r_m^0 \), \( W_s^0 = r_s^m \), \( \Sigma_h d\omega^h = 0 \). This perturbation has the properties that \( H - 1 \) portfolios can be arbitrarily controlled without affecting allocations \((x, y)\), prices \( q \), and also without changing \( \zeta \). Indeed if normalized state prices \( \frac{\lambda^h}{\lambda_0^j} \) are not identical across agents, this perturbation can be used to locally control \( \frac{\zeta}{\omega^h} \Sigma_h \theta_m^h \lambda^h \frac{\lambda}{\lambda_0^j} \), without affecting the equilibrium conditions. Since the property that \( \frac{\zeta}{\omega^h} \Sigma_h \theta_m^h \lambda^h \frac{\lambda}{\lambda_0^j} \neq 0 \) is open, we can conclude that there exists a generic set of endowments \( \Omega^* \) such that for every \( \omega \) in \( \Omega^* \), at equilibrium, \( \frac{\zeta}{\omega^h} \Sigma_h \theta_m^h \lambda^h \frac{\lambda}{\lambda_0^j} \neq 0 \). We are only left to show that \( \lambda^h \lambda^h_0 \neq \lambda^h_0 \) for some \( h, h' \) in \( \{ h : \theta_m^h > 0 \} \). But this is a well known, standard, property in GEI that immediately generalizes to our economy. By projecting the set of economies for which equilibria satisfy \( \Sigma_h \theta_m^h \lambda^h \frac{\lambda}{\lambda_0^j} \neq 0 \) onto \( \Omega \), we obtain \( \Omega^* \). \( \square \)

Observe that unlike in a standard GEI model, constrained inefficiency can be proved for fixed preferences, and is therefore generic in the sense that it holds for an open an full Lebesgue measure set of economies. In GEI this was not possible because constrained suboptimality would depend on the indirect, income, effects caused by a portfolios redistribution. These effects vanish if utilities are (even locally) homothetic; this is where perturbations of utilities were needed. Instead, in our model welfare improvements of equilibria can be attained through first order effects: the externalities pointed out above, commenting on the planner’s problem. To understand better these effects, it is instructive to isolate them from the other sources of inefficiencies which are more generally accruing due to market incompleteness. In the rest of the section, we consider two benchmark models in which, without information asymmetries, equilibria deliver constrained efficient allocations. Precisely, for productive inefficiencies we appeal to an example of an economy in which two firms have technologies characterized by ‘multiplicative uncertainty’, as in Diamond (1967). While for financial inefficiencies we present an example based on the CAPM.

**Example 1** (productive inefficiency) Suppose that in our economy two firms, that we label \( j = L, H \), have the following very simple technologies, characterized by ‘multiplicative uncertainty’ (Diamond 1967):
\[
\begin{align*}
Y^L &= \left\{ y \in \mathbb{R}^{S+1} \mid y = (-y_0, \ldots, \phi_s f^L(y_0), \ldots) \right\} \\
Y^H &= \left\{ y \in \mathbb{R}^{S+1} \mid y = (-y_0, \ldots, \phi_s f^H(y_0), \ldots) \right\}.
\end{align*}
\]
Suppose further that the first technology is less efficient than the second, \( f^L(\cdot) = \alpha f^H(\cdot) = \alpha f(\cdot) \), with \( 0 < \alpha < 1 \) and \( f : \mathbb{R} \to \mathbb{R} \) strictly increasing and strictly concave. According to Definition 5, first order conditions for productive efficiency imply
\begin{equation}
\frac{\partial f(y^H_0)}{\partial y^H_0} = \alpha \frac{\partial f(y^L_0)}{\partial y^L_0}
\end{equation}

which requires \( y^H_0 > y^L_0 \). To understand this condition, observe that operating both technologies, rather than one, does not provide any advantage in terms of risk sharing: once the initial level of investments have been chosen, the second period returns, \((y^L_1, y^H_1)\), are collinear. Condition (5) entails equality of marginal productivity of investment in the two firms.

We now argue that productive efficiency cannot be achieved at equilibrium. Indeed, suppose that investors cannot distinguish between the two firms, and that firms obtain outside financing through the fund market. This implies that firms \( H \) and \( L \) belong to the same sector \( m \) with price \( q^m \) and returns \( r^m_s \).

Assume each of the two firms \( j = L, H \) belongs to a single owner, \( h = j \). The single owner’s choices of \( \zeta^j \) and \( y^j_0 \) satisfy the first order conditions:

\[ \zeta^j : \quad \partial u^j_0(y^j_0 + q^m) - \sum_s \partial u^j_s \phi^j f^j(y^j_0) = 0 \]

\[ y^j_0 : \quad (1 - \zeta^j) \left[ \partial u^j_0 - \sum_s \partial u^j_s \phi^j \frac{\partial f^j(y^j_0)}{\partial y^j_0} \right] = 0 \]

leading to

\[ \sum_s \frac{\partial u^j_s}{\partial u^j_0} \phi^j = \frac{y^j_0 + q^m}{f^j(y^j_0)} = \frac{1}{\frac{\partial f^j(y^j_0)}{\partial y^j_0}}. \]

Using the properties of the technologies described above, the last equality leads to \( y^H_0 = y^L_0 \): with respect to the efficient allocation of inputs, incomplete information leads to overinvestment in the "lemon" project \( j = L \), relative to the investment in the good project \( j = H \).

**Example 2** (financial inefficiency) Next, we would like to present an example showing that in our model efficiency may be impaired by financial decisions. To do so we consider a CAPM economy. Initial endowments, \( \omega \), are deterministic, and preferences are mean-variance, defined over date 1 consumption by \( u^h(E(x_1), Var(x_1)) \), with \( u^h \) monotonically increasing in the mean term and decreasing in the variance; where, hereafter, all statistical moments are defined as weighted averages of the variables across \( s \), with weights equal to the common prior probabilities of the events \( s \) in \( S \). Finally, in line with the general model, we assume that the only risky asset traded in the economy is a single mutual fund with price \( q \) and return \( r \).

Since endowments are deterministic, individuals face two sources of uncertainty: one due to portfolio holdings and the other due to firm ownership. Efficient risk sharing is obtained when the latter risk is completely hedged, i.e., when \( \zeta^j = 1 \) for all \( j \). Indeed, when \( \zeta^j = 1 \), individual optimality yields the two-fund-separation
property. Since this is true for all \( j \), \( r_s^m = \frac{1}{J} \sum_{j \in m} y_j^j \), i.e., the fund coincides with 'the market'. In other words, when endowments are deterministic, the first best allocation is constrained feasible. Moreover, if consumers/entrepreneurs have identical preferences, and their feasible investment projects have identical variance, \( \sigma \), then an efficient (first best) allocation is attained when \( \theta^j = 1 \), and \( \zeta^j = 1 \), for all \( j \). At this allocation the variance of holding the fund, \( \sigma_r \), and the agents’ portfolio variances, are minimized.\(^8\)

To show financial inefficiency, we argue that the first best allocation of this economy is not achievable as a competitive equilibrium.

For expositional purpose, suppose that each entrepreneur \( j \) has endowments \( (\omega_0^j, \omega_1^j) = (1, 0) \), and that the economy has only two firms, \( j = L, H \), and one sector. Moreover, let each technology \( j \) consist of a unique production plan with a fixed activation cost, \( y_0^j = 1 \), and a stochastic return \( y_1^j \) having mean \( \mu^j \) and variance \( \sigma \). Assume that \( \mu^H = \mu, \mu^L = \alpha \mu, \alpha \in (0, 1) \), and that the covariance among the returns from the two projects be equal to \( c \). Then, the date zero budget constraint of \( j \) is \( \theta_0^j = (q + 1) (\zeta^j - \theta^j) \). Using this in the second period budget constraint yields \( x_1^j = (1 - \zeta^j) y_1^j + r_1 \theta^j + (q + 1) (\zeta^j - \theta^j) \). From this it is straightforward to compute the mean and the variance of \( x_1^H \). Next, consider the following marginal, financial, reallocation for \( H \): \( d \zeta^H = d \theta^H \). This is feasible and implies \( d \theta_0^H = 0 \). Evaluating the effects of this change on \( H \)'s expected utility yields:

\[
\left( \frac{du^H}{d \zeta^H} \right)_{\zeta=\theta=1} = (u_1[-\mu + E(r_1)] + u_2[-2(1 - \zeta^H)\sigma + 2 \theta^H \sigma_r + 2 \text{Cov}(y^H, r)((1 - \zeta^H) - \theta^H)])_{\zeta=\theta=1} = u_1 \left[ -\mu + \mu (1 + \alpha) \right] = -u_1 \left[ \frac{(1 - \alpha)}{2} \mu \right] < 0
\]

where \( u_1 > 0, u_2 < 0 \) are the partial derivatives of \( u \) with respect to the mean and the variance of \( H \)'s consumption; and where at \( \zeta = \theta = 1 \), \( \text{Cov}(y^H, r) = \sigma_r = (\sigma + c)/2 \). Thus, by lowering \( \zeta^H \) below 1, \( H \) increases her expected utility. Intuitively, the resulting inefficiency is due to the fact that \( H \) does not take into account the implications of \( d \zeta^H \) on the mean and variance of the funds return \( r \). At equilibrium, \( \zeta^H / \bar{c} < 1/2 \), denoting an overrepresentation in the fund portfolio of the ‘lemon’ project, \( j = L \), relative to the good project, \( j = H \).

The previous two examples illustrate the inefficiencies due the inability of the firms to take into account the effect on the fund’s return when choosing their initial investment or financial policy. Going back to (2), one sees that another source of inefficiency may come from the choice of the state distribution of profits (in Example 1 technology is multiplicative, while in Example 2 production plans are fixed, so this additional effect is absent).

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\(^8\) When there are only two projects, \( j = H, L \), with identical variance, \( \sigma \), then \( \sigma_r = (\delta^2 + (1 - \delta)^2)\sigma + 2\delta(1 - \delta)c \), where, \( \delta = \zeta^H / (\zeta^H + \zeta^L) \). \( \sigma_r \) attains a unique minimum at \( \delta^* = 1/2 \) when \( c \neq \sigma \). At \( \delta = 1/2 \), \( \sigma_r = (\sigma + c)/2 \).
Example 3 (inefficient state-distribution of profits) We are now going show that production inefficiency may be induced by an ‘incorrect’, state-distribution of the returns from production.

For simplicity, assume that consumers, who are not entrepreneurs, have quadratic utilities,  
\[ u^h(x) = \sum_{s>0} \rho_s (x_s - \frac{1}{2} (c-x_s)^2), \]
where \( c = \max_{s \in S} (\omega_s = \sum_h \omega^h_s) \). Further, assume that there is only one sector in which -among the others- there is an entrepreneur, \( j \), who is risk neutral, with  
\[ u_j(x) = a_j \sum_{s>0} \rho_s x_s. \]

Then, every equilibrium with \( (y^j, \xi^j) \gg 0 \) is constrained inefficient. Indeed, suppose that the planner decides a marginal change in the production of  
\[ dy^j = (dy^{j}_0, dy^{j}_1), \]
that is feasible (i.e.,  
\[ DF_j dy^j = 0. \]
Since the initial production allocation was optimal for  
\( j \),  
\[ dy^j_0/q^0 \]
has no effect on the firm profits, implying  
\[ dy^j_0/q^0 = \frac{E(dy^j_1)}{\xi}. \]
Moreover, the planner can finance this change in production at  
\( t=0 \), by redistributing income from date 1 to date 0, and from the consumers to the firm, using the riskless bond. Since these redistributions are feasible, they can be decentralized at the original equilibrium price (say  
\( q^0 \)), so that they have no effects on welfare too.

However, if the covariance of  
\( dy^j \) with respect to consumers’ income (or consumption) is negative, then  
\( dy^j \) is Pareto improving. To see the latter, let  
\[ \lambda^h = Du^h, \]
and observe that  
\[ \sum_h \sum_{s>0} \theta^h \lambda^h_s dy^{j(-)}_s \xi^j = \sum_h \sum_{s>0} \theta^h \left( (c' - E(x^h)) E(dy^{j(-)}_s) - Cov(x^h, dy^j) \right) \xi^j \]
\[ = - \sum_h \theta^h Cov(x^h, dy^j) \xi^j \]
where in the second row we exploited the property that for each pair of random variables  
\( X, Y, E(XY) = Cov(X, Y) + E(X)E(Y); \) while in the third row we used the fact that,  
\[ E(dy^{j(-)}) = -dy^{j}_0/q^0 + E(dy^{j}_1) = 0. \]

The intuition behind this result is that the insurance (or covariance) effect is not anticipated by competitive agents, who take their investment decisions at a given  
\( (q, r) \). Here, this is true a fortiori because  
\( j \)—being risk neutral—has not even personal benefits from the risk sharing effects of the state-distribution of the returns from production.

3 Active funds

So far we have argued that in our economy prices fail to coordinate agents to the achievement of a CPO. The examples presented in the previous section show that,
at equilibrium, first order conditions for CPO are not satisfied. A second problem, emerging from the examples, concerns the possibility of decentralizing a CPO allocation as an equilibrium of our economy. In Example 2, we effectively proved constrained inefficiency of equilibria by noticing that the CPO allocation would not satisfy some individual optimality conditions. A similar result can be easily derived for the first example.9

The difficulty in decentralizing CPO allocations is specific to our economy, and does not arise in standard GEI models. The fund structure reduces the number of prices that can be used to decentralize allocations.

In this section we propose a modification of our model that, by recognizing a more active role to funds, allows to overcome inefficiencies, and circumvents the decentralization problem.

Indeed, consider an economy $E$ in which funds actively represent the interests of their owners at the shareholders’ meetings of the firms which they finance: each fund $m$ votes on the choice of the production plan of every firm $j$ in its sector; and at the same time $m$ negotiates a firm specific price, $q^j = (q^m + \tau^j)$, for the financing of $j$. The objective of the firm must now take into account the interests of the fund owners. Following Drèze (1974), we let the firm choose production such as to maximize $\pi^j y^j$ where $\pi^j = (1 - \xi^j) \frac{\lambda^s_j}{\lambda^0} + \xi^j \pi^m_s$, and $\pi^m$ is the vector of fund’s state prices:

$$\pi^m = \sum_h \frac{\theta^h m}{\theta^m} \lambda^h_s \lambda^0_s.$$

The relevant first order conditions are:

$$y^j = \frac{1}{1 - \xi^j} \lambda^j_s \lambda^0_s + \xi^j \pi^m_s, \quad s \in S$$

(6)

$$\xi^j \cdot q^m + y^j + \tau^j - \sum_{s>0} \lambda^j_s \lambda^0_s y^j = 0$$

where $\nabla_s F^j = \partial_s F^j / \partial_0 F^j$.

Let the transfer $\tau^j$ be such as to cover the discrepancy between the market valuation of the production plan and its value in the eyes of the fund: $\tau^j = \sum_{s \geq 0} \pi^m_s y^j - q^m$. Then, using the expressions for the derivatives of the fund’s return with respect to $y^j$ and $\xi^j$, one sees that the first order conditions in (6) coincide with the corresponding ones in (2):10

$$y^j = \frac{1}{1 - \xi^j} \lambda^j_s \lambda^0_s + \frac{1}{\lambda^0_s} \sum_h \frac{\theta^h m}{\theta^m} \lambda^h_s \xi^j, \quad s \in S$$

9 Take the function $f(\cdot)$ to be logarithmic, with $y_0 \geq 1$. Suppose that one tries to implement $y^H_0 > y^L_0$, say $y^L_0 = \beta y^H_0$, $y^H_0 = y_0$, $0 < \beta < 1$. Then at equilibrium first order conditions imply $y_0 (1 - \beta) = -y_0 \log \beta$, which does not hold for any $y_0 > 0$, if $0 < \beta < 1$.

10 In this comparison it is important to recall that $\lambda^h_s = \alpha^h \lambda^h_0$, $\lambda^h_s = \alpha^h \lambda^h_0$, (implying $\hat{\alpha} = \lambda^h_s / \lambda^h_0$) for all $h$, and all $s \in S$; where the variables with a ‘hat’ are the Lagrange multipliers of the CPO program. Finally, one can use the following results: $\partial r^m / \partial y^j_s = \xi^j / \xi^m$, $\partial r^m / \partial \xi^j = (y^j - r^m) / \xi^m$, $\xi^m = \bar{\theta}_m$. 

\[ \zeta^j : q^m + y_0^j - \sum_{s > 0} \lambda_s^j y_s^j + \frac{q^m}{\lambda_0} \sum_{h \geq 0} \sum_{s > 0} \frac{\partial h^m}{\partial m} \lambda_s^h (y_s^j - r_m^s) = 0, \quad j \in J, \; m \in M. \]

The active role of the fund leads firm \( j \) to internalize the externality. Moreover, the transfers \( \tau^j \) add up to zero for the fund:

\[ \sum_{j \in m} \tau^j \zeta^j = \sum_{s \geq 0} \pi_s^m \sum_{j \in m} \zeta^j y_s^j - q^m \bar{\zeta}^m = \left[ \sum_{s \geq 0} \pi_s^m r_s^m - q^m \right] \bar{\zeta}^m = 0. \]

This construction assumes that the funds (a) take an active role in the production decisions of the firms they finance, (b) know the preferences of their shareholders and act on their behalf, (c) are allowed to share the information of the initial owners when contracting with them on the transfer \( \tau \). These are fairly demanding assumptions. Yet our observation may be of some practical interest, since it suggests a reduced form model of venture capital (see, for example, Gompers and Lerner 1999).

References


