

# Contents

<b>I</b>	<b>Programming techniques for financial calculus</b>	<b>9</b>
<b>1</b>	<b>An introduction to MATLAB<sup>®</sup> with applications</b>	<b>11</b>
1.1	MATLAB <sup>®</sup> basics . . . . .	11
1.1.1	Preliminary elements . . . . .	12
1.1.2	Vectors and matrices . . . . .	15
1.1.3	Basic linear algebra operations . . . . .	18
1.1.4	Element-by-element multiplication and division . . . . .	21
1.1.5	Colon (:) operator . . . . .	22
1.1.6	Predefined and user-defined functions . . . . .	24
1.2	M-file: Scripts and Functions . . . . .	26
1.3	Programming fundamentals . . . . .	29
1.3.1	if, else, and elseif construct . . . . .	29
1.3.2	for loops . . . . .	32
1.3.3	while loops . . . . .	33
1.4	MATLAB <sup>®</sup> graphics . . . . .	34
1.5	Preliminary exercises on programming . . . . .	36
1.6	Exercises on the basics of financial evaluation . . . . .	49
1.6.1	Interest Rate Swap . . . . .	55
<b>II</b>	<b>Portfolio selection</b>	<b>61</b>
<b>2</b>	<b>Preliminary elements in Probability Theory and Statistics</b>	<b>63</b>
2.1	Basic concepts in probability . . . . .	63
2.2	Random variables . . . . .	71
2.3	Probability distributions . . . . .	74
2.4	Continuous random variables . . . . .	76
2.5	Higher-order moments and synthetic indices of a distribution . . . . .	80
2.6	Some probability distributions . . . . .	82

2.6.1	Uniform distribution . . . . .	83
2.6.2	Normal distribution . . . . .	86
2.6.3	Log-normal distribution . . . . .	91
2.6.4	Chi-square distribution . . . . .	94
2.6.5	Student-t distribution . . . . .	95
<b>3</b>	<b>Linear and Non-linear Programming</b>	<b>99</b>
3.1	General Framework . . . . .	99
3.2	Optimization with MATLAB® . . . . .	100
3.2.1	Linear Programming . . . . .	101
3.2.2	Quadratic Programming . . . . .	102
3.2.3	Non-Linear Programming . . . . .	104
3.3	Multi-objective optimization . . . . .	106
3.3.1	Efficient solutions and the efficient frontier . . . . .	108
<b>4</b>	<b>Portfolio Optimization</b>	<b>111</b>
4.1	Portfolio of equities: prices and returns . . . . .	112
4.2	Risk-return analysis . . . . .	116
4.2.1	Elements of Expected Utility Theory . . . . .	116
4.2.2	General Framework . . . . .	118
4.2.3	Mean-Variance model . . . . .	119
4.2.4	Effects of diversification for an EW portfolio . . . . .	139
4.2.5	Mean-Mean Absolute Deviation model . . . . .	141
4.2.6	Mean-Maximum Loss model . . . . .	145
4.2.7	Value-at-Risk . . . . .	149
4.2.8	Mean-Conditional Value-at-Risk model . . . . .	152
4.2.9	Mean-Gini model . . . . .	161
4.3	Elements of bond portfolio immunization . . . . .	166
<b>III</b>	<b>Derivatives pricing</b>	<b>175</b>
<b>5</b>	<b>Further elements on Probability Theory and Statistics</b>	<b>177</b>
5.1	Introduction to Monte Carlo simulation . . . . .	177
5.2	Stochastic processes . . . . .	179
5.2.1	Brownian motion . . . . .	187
5.2.2	Ito's Lemma . . . . .	192
5.2.3	Geometric Brownian motion . . . . .	196

<b>6 Pricing of derivatives with an underlying security</b>	<b>199</b>
6.1 Binomial model . . . . .	200
6.1.1 A replicating portfolio of stocks and bonds . . . . .	201
6.1.2 Calibration of the binomial model . . . . .	206
6.1.3 Multi-period case . . . . .	209
6.2 Black-Scholes model . . . . .	213
6.2.1 Assumptions of the model . . . . .	214
6.2.2 Pricing of a European call . . . . .	218
6.2.3 Pricing equation for a call . . . . .	220
6.2.4 Implied volatility . . . . .	223
6.2.5 Black-Scholes formulas via integrals . . . . .	224
6.3 Option Pricing via the Monte Carlo method . . . . .	226
6.3.1 Path Dependent Derivatives . . . . .	228
<b>References</b>	<b>229</b>
<b>Suggested lesson plan</b>	<b>235</b>