# Methods for Constructing Non-compensatory Composite Indices: A Comparative Study

Matteo Mazziotta and Adriano Pareto

**Abstract** Non-compensability and comparability of the data over time are central issues in composite indices construction. The aim of this paper is to compare two non-additive approaches: Mazziotta-Pareto Index (MPI) and Weighted Product (WP) method. The MPI is a non-linear composite index which rewards the units with 'balanced' values of the individual indicators. It normalizes the components with respect to the mean and standard deviation and allows relative comparisons only. The WP method allows to build, for each unit, two composite indices closely interrelated: a 'static' index for space comparisons, and a 'dynamic' index for time comparisons. An application to indicators of well-being in the Italian regions in 2005 and 2009 is presented

**Key words:** composite indices, normalization, aggregation

## 1 Introduction

Composite indices for comparing country performance with respect to multi-dimensional phenomena, such as development, poverty, quality of life, etc., are increasingly recognized as a useful tool in policy and public communication (OECD, 2008).

Considerable attention has been devoted in recent years to the fundamental issue of compensability among the components of the index (a deficit in one dimension can be compensated by a surplus in another) and more and more often a non-compensatory approach has been adopted (e.g. the 'new' Human Development Index calculated by UNDP in 2010 is given by a geometric mean).

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In this work, we compare two different non-additive approaches: Mazziotta-Pareto Index (MPI) and Weighted Product (WP) method.

The MPI is a non-linear composite index which transforms the individual indicators in standardized variables and summarizes the data using an arithmetic mean adjusted by a ‘penalty’ coefficient related to the variability of each unit. The aim is to penalize the units with ‘unbalanced’ values of the indicators in a non-compensatory perspective.

The WP method, also termed as the geometric aggregation approach, is a classic data aggregation technique in index number theory. An application of the Jevons index to indicized indicators is presented that allows to build, for each unit, a ‘static’ and a ‘dynamic’ index, for both spatial and temporal comparisons.

In Section 2 a brief description of the MPI is reported; in Section 3 the indices based on the WP method are presented; finally, in Section 4 an application to real data is proposed.

## 2 Mazziotta-Pareto Index

The Mazziotta-Pareto Index is a composite index based on the assumption of ‘non-substitutability’ of the indicators, i.e., they have all the same importance and a compensation among them is not allowed (De Muro *et al.*, 2010).

The index is designed in order to satisfy the following properties: (i) normalization of the indicators by a specific criterion that deletes both the unit of measurement and the variability effect; (ii) synthesis independent from an ‘ideal unit’, since a set of ‘optimal values’ is arbitrary, non-univocal and can vary with time; (iii) simplicity of computation; (iv) ease of interpretation.

Let us consider a set of individual indicators positively related with the phenomenon to be measured. Given the matrix $\mathbf{X}=\{x_{ij}\}$ with $n$ rows (units) and $m$ columns (indicators), we calculate a standardized matrix $\mathbf{Z}=\{z_{ij}\}$ as follow:

$$z_{ij} = 100 + \frac{(x_{ij} - \mathrm{M}_{x_j})}{\mathrm{S}_{x_j}} 10$$

where $\mathrm{M}_{x_j}$ and $\mathrm{S}_{x_j}$ are, respectively, the mean and the standard deviation of the $j$-th indicator.

Denoting with $\mathrm{M}_{z_i}$ and $\mathrm{S}_{z_i}$, respectively, the mean and the standard deviation of the standardized values of the $i$-th unit, the generalized form of MPI is given by:

$$\mathrm{MPI}_i^{+/-} = \mathrm{M}_{z_i} \pm \mathrm{S}_{z_i} \mathrm{cv}_i$$

where $\mathrm{cv}_i = \mathrm{S}_{z_i} / \mathrm{M}_{z_i}$ is the coefficient of variation of the $i$-th unit and the sign ± depends on the kind of phenomenon to be measured.

If the composite index is ‘increasing’ or ‘positive’, i.e., increasing values of the index correspond to positive variations of the phenomenon (e.g., the socio-economic development), then $\mathrm{MPI}^-$ is used. Vice versa, if the composite index is ‘decreasing’ or ‘negative’, i.e., increasing values of the index correspond to negative variations of the phenomenon (e.g., the poverty), then $\mathrm{MPI}^+$ is used (Mazziotta and Pareto, 2011).

This approach is characterized by the use of a function (the product $S_{z_i} cv_i$) to penalize the units with 'unbalanced' values of the indicators. The 'penalty' is based on the coefficient of variation and is zero if all the values are equal. The purpose is to favour the units that, mean being equal, have a greater balance among the different indicators.

# 3 'Static' and 'dynamic' composite index

The weighted product method is one of the major techniques in composite index construction since it represents a trade-off solution between additive methods with full compensability and non-compensatory approaches (OECD, 2008).

When an unweighted geometric mean of ratios - such as the Jevons index - is computed, the obtained result satisfies many desirable properties from an axiomatic point of view (Diewert, 1995).

Let $x_{ij}^t$ the value of the indicator *j* for the region *i* at time *t* ($j$=1, …, $m$; $i$=1, …, $n$; $t$=$t_0$, $t_1$). A 'static' composite index may be defined as follows:

$$\mathrm{SCI}_i^t = \prod_{j=1}^{m} \left( \frac{x_{ij}^t}{x_{rj}^t} 100 \right)^{\frac{1}{m}}$$

where $x_{rj}^t$ is the reference or base value, e.g., the average. Therefore, values of SCI that are higher (lower) than 100 indicate regions with above (below) average performance.

In order to compare the data from time $t_0$ to $t_1$, for each unit, we can construct a 'dynamic' composite index given by:

$$\mathrm{DCI}_i^{t_1/t_0} = \prod_{j=1}^{m} \left( \frac{x_{ij}^{t_1}}{x_{ij}^{t_0}} 100 \right)^{\frac{1}{m}} .$$

For the 'circularity' or 'transitivity' property of the index number theory, SCI and DCI are linked by the relation: $\mathrm{DCI}_i^{t_1/t_0} = (\mathrm{SCI}_i^{t_1} / \mathrm{SCI}_i^{t_0}) \, \mathrm{DCI}_r^{t_1/t_0}$ .

# 4 An application to the Italian regions

In order to compare the two approaches, we consider a set of indicators of well-being in the Italian cities, at regional level, in 2005 and 2009.

The variables used are the following: Sporting activities, Close to supermarkets, Green space, Public transport, Parking provision, Children's services, Elderly home care. The $\mathrm{MPI}^-$ is used, since the composite index is 'positive', i.e., increasing values of the index correspond to positive variations of well-being.

Data matrix is reported in Table 1 and results are presented in Table 2.

**Table 1:** Individual indicators of well-being in the Italian regions (Years 2005, 2009)

| Regions | 2005 | | | | | | | 2009 | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Sporting activities | Close to supermarkets | Green space | Public transport | Parking provision | Children's services | Elderly home care | Sporting activities | Close to supermarkets | Green space | Public transport | Parking provision | Children's services | Elderly home care |
| Piemonte | 34.1 | 60.3 | 42.0 | 189.8 | 12.5 | 28.6 | 1.8 | 34.1 | 69.0 | 42.5 | 199.3 | 17.1 | 37.1 | 2.3 |
| Valle d'Aosta | 33.9 | 52.7 | 23.2 | 544.0 | 5.3 | 100.0 | 0.1 | 46.3 | 58.6 | 26.2 | 580.0 | 8.4 | 78.4 | 0.4 |
| Lombardia | 37.7 | 69.9 | 27.6 | 230.1 | 20.0 | 54.6 | 3.2 | 36.5 | 68.9 | 28.6 | 227.7 | 24.1 | 62.5 | 4.1 |
| Trentino-Alto Adige | 53.1 | 72.2 | 71.2 | 190.7 | 28.6 | 75.8 | 0.6 | 48.2 | 71.9 | 70.3 | 192.9 | 34.5 | 83.8 | 0.8 |
| Veneto | 39.4 | 65.8 | 58.7 | 122.5 | 39.8 | 42.7 | 5.0 | 39.6 | 70.1 | 62.3 | 124.4 | 42.2 | 70.2 | 4.8 |
| Friuli-Venezia Giulia | 36.7 | 72.7 | 21.8 | 257.4 | 11.9 | 53.0 | 7.9 | 37.5 | 74.6 | 22.1 | 258.1 | 12.0 | 83.6 | 7.7 |
| Liguria | 26.6 | 67.9 | 35.3 | 312.5 | 23.1 | 75.3 | 3.1 | 27.6 | 70.6 | 35.4 | 311.0 | 22.3 | 64.3 | 3.4 |
| Emilia-Romagna | 32.4 | 71.1 | 158.5 | 81.0 | 24.4 | 78.0 | 5.4 | 36.8 | 69.3 | 157.7 | 83.0 | 24.0 | 88.0 | 8.3 |
| Toscana | 30.4 | 68.7 | 152.5 | 106.0 | 18.6 | 78.0 | 2.1 | 33.1 | 64.3 | 152.1 | 108.4 | 20.9 | 74.6 | 2.2 |
| Umbria | 31.2 | 65.9 | 192.1 | 162.4 | 27.4 | 51.1 | 4.1 | 32.3 | 73.7 | 187.6 | 162.8 | 26.9 | 63.0 | 7.6 |
| Marche | 31.4 | 76.0 | 185.8 | 157.2 | 9.2 | 45.9 | 3.3 | 32.2 | 67.4 | 186.1 | 157.7 | 15.3 | 55.7 | 3.6 |
| Lazio | 33.7 | 74.3 | 127.4 | 124.5 | 6.5 | 30.4 | 3.3 | 29.4 | 74.7 | 121.0 | 132.3 | 7.0 | 30.7 | 4.0 |
| Abruzzo | 28.9 | 55.5 | 714.5 | 93.5 | 5.3 | 26.2 | 1.8 | 31.0 | 63.0 | 710.0 | 93.5 | 21.1 | 52.1 | 4.8 |
| Molise | 23.2 | 52.1 | 18.3 | 177.2 | 1.3 | 2.9 | 6.1 | 22.0 | 58.7 | 18.5 | 177.2 | 1.2 | 7.4 | 2.4 |
| Campania | 22.3 | 59.3 | 24.8 | 227.3 | 7.3 | 39.2 | 1.4 | 21.1 | 60.0 | 25.9 | 218.0 | 5.9 | 50.5 | 1.9 |
| Puglia | 25.8 | 70.3 | 7.8 | 114.3 | 7.3 | 27.5 | 2.0 | 23.8 | 69.6 | 8.1 | 122.0 | 8.2 | 44.2 | 2.0 |
| Basilicata | 24.4 | 55.5 | 547.9 | 84.9 | 2.4 | 32.8 | 3.9 | 27.1 | 65.2 | 545.6 | 87.4 | 2.3 | 21.4 | 5.1 |
| Calabria | 24.5 | 55.1 | 19.7 | 159.6 | 20.3 | 7.8 | 1.6 | 24.8 | 56.4 | 20.8 | 172.8 | 19.5 | 15.6 | 2.5 |
| Sicilia | 21.5 | 63.6 | 71.5 | 72.2 | 3.4 | 33.3 | 0.8 | 22.5 | 68.6 | 73.3 | 75.7 | 6.5 | 34.6 | 1.1 |
| Sardegna | 31.1 | 75.9 | 86.4 | 55.7 | 16.8 | 17.2 | 1.1 | 28.2 | 78.3 | 85.9 | 56.6 | 16.9 | 20.4 | 2.3 |
| **Italia** | **31.3** | **67.1** | **93.5** | **118.8** | **14.4** | **42.8** | **2.9** | **31.1** | **68.5** | **93.6** | **122.1** | **16.2** | **51.7** | **3.6** |

Source: http://www3.istat.it/ambiente/contesto/infoterr/assi/asseV.xls

**Table 2:** MPI, SCI and DCI of well-being (Years 2005, 2009)

| Region | Mazziotta-Pareto Index | | | Weighted Product method | | |
|---|---|---|---|---|---|---|
| | $MPI_{05}$ | $MPI_{09}$ | $MPI_{09}$-$MPI_{05}$ | $SCI_{05}$ | $SCI_{09}$ | $DCI_{09/05}$ |
| Piemonte | 97.0 | 98.6 | 1.6 | 82.1 | 87.5 | 115.6 |
| Valle d'Aosta | 99.0 | 100.1 | 1.1 | 63.8 | 75.5 | 128.4 |
| Lombardia | 103.4 | 102.9 | -0.6 | 105.0 | 104.5 | 107.9 |
| Trentino-Alto Adige | 106.0 | 105.3 | -0.7 | 106.9 | 105.1 | 106.7 |
| Veneto | 104.5 | 105.5 | 1.1 | 120.9 | 122.5 | 110.0 |
| Friuli-Venezia Giulia | 105.6 | 106.2 | 0.6 | 108.6 | 107.5 | 107.4 |
| Liguria | 103.3 | 102.0 | -1.3 | 114.7 | 105.3 | 99.6 |
| Emilia-Romagna | 104.9 | 105.9 | 1.0 | 132.7 | 134.3 | 109.9 |
| Toscana | 101.2 | 99.5 | -1.8 | 113.4 | 107.3 | 102.6 |
| Umbria | 103.3 | 106.2 | 2.8 | 136.6 | 143.6 | 114.1 |
| Marche | 101.6 | 100.3 | -1.3 | 113.1 | 115.0 | 110.3 |
| Lazio | 99.4 | 97.8 | -1.6 | 93.4 | 87.8 | 102.0 |
| Abruzzo | 96.0 | 102.2 | 6.1 | 93.5 | 137.3 | 159.3 |
| Molise | 91.9 | 89.1 | -2.8 | 41.6 | 38.5 | 100.5 |
| Campania | 94.3 | 92.7 | -1.6 | 68.4 | 65.9 | 104.5 |
| Puglia | 95.2 | 94.5 | -0.8 | 55.3 | 55.6 | 109.1 |
| Basilicata | 95.9 | 96.4 | 0.5 | 89.7 | 83.7 | 101.3 |
| Calabria | 93.1 | 92.0 | -1.1 | 59.5 | 65.7 | 119.8 |
| Sicilia | 92.0 | 92.4 | 0.4 | 54.9 | 60.0 | 118.5 |
| Sardegna | 96.7 | 96.5 | -0.2 | 70.4 | 73.6 | 113.5 |

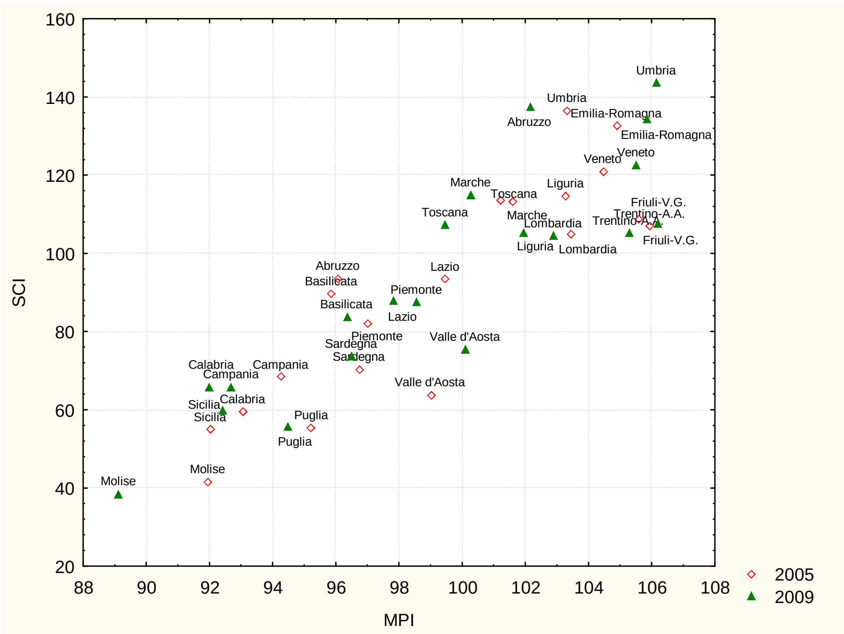
Note that the base value of the ‘static’ indices ($SCI_{05}$ and $SCI_{09}$), for each region, is the national average (Italy), while the base of the ‘dynamic’ index ($DCI_{09/05}$) is the value for the year 2005.

As we can see from Table 2, not necessarily each relative increase corresponds to an absolute one and vice versa. For example, from 2005 to 2009, Toscana shows a reduction of the level of well-being compared to the average ($MPI_{09}$-$MPI_{05}$=-1.8; $SCI_{05}$=113.4 vs. $SCI_{09}$=107.3), though the values of the individual indicators, on the whole, are increased ($DCI_{09/05}$=102.6). This is due to a greater rise of the performances of the other regions which has produced a large increase of the national average in 2009.

Overall, the region in which it is possible to record the highest increase of the well-being indicators, over the five years, is Abruzzo ($MPI_{09}$-$MPI_{05}$=+6.1; $DCI_{09/05}$=159.3). From the point of view of the decrease, instead, the results are conflicting: the greatest relative decrease, in fact, is for Molise ($MPI_{09}$-$MPI_{05}$=-2.8), although the values of the individual indicators are, on average, slightly increased ($DCI_{09/05}$=100.5); while the largest absolute decrease is observed in the Liguria Region ($DCI_{09/05}$=99.6).

In Figure 1, the comparison between the scores obtained by MPI and SCI is presented, for the year 2005 (ρ=0.84) and the year 2009 (ρ=0.88); in general the results are concordant and the main differences are due to the different method of normalization of the two methodologies. The MPI, in fact, assigns the same weight to all the components, while the SCI assigns different weights depending on the variability.

**Figure 1:** Comparing MPI and SCI (Years 2005, 2009)



# 5 Conclusions

Non-compensability and comparability of the data over time are central issues in composite indices construction. Non-compensatory composite indices may be obtained by non-additive approaches; while the question of comparability mainly depends on the normalization method.

In this paper, a comparison between two different approaches is proposed. The MPI is based on a standardization with respect to the mean and standard deviation that makes the indicators independent of the variability. Therefore, all the variables are assigned equal weights, and only relative time comparisons are allowed.

The two indices based on the WP method implicitly give more weight to the components that exhibits the largest variability, and the DCI allows absolute time comparisons too.

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