CORRELATION
In today’s class…

Cross-Correlation
  Concepts
  Relation with Convolution
  Examples
Auto-Correlation
Correlation

Definition:

Correlation or “Co-Relation” is a measure of similarity/relationship between two signals.

If \(x[n]\) & \(y[n]\) are two discrete-time signals, then the correlation of \(x[n]\) with respect to \(y[n]\) is given as,

\[
r_{xy}[l] = \sum_{m=-\infty}^{\infty} x[m] y[m - l]
\]

where \(l\) is lag, indicating time-shift.
Applications of Correlation

There are applications where it is necessary to compare one reference signal with one or more signals to determine the similarity between the pair and to determine additional information based on the similarity.
For example, in digital communications, a set of data symbols are represented by a set of unique discrete-time sequences.

If one of these sequence has been transmitted, the receiver has to determine which particular sequence has been received, by comparing the received signal with every member of possible sequences from the set.

Similarly correlation can also be used to timing recovery purpose.

Also used in CDMA receivers.
Computing Cross-Correlation

Cross-Correlation: Correlation between two different sequences
Matlab command \texttt{xcorr()} can be used for this purpose

\[
x - \text{corr } x \text{ wrt } y \\
\begin{align*}
r_{xy}[l] &= \sum_{m=-\infty}^{\infty} x[m] y[m - l] \\
r_{xy}[l] &= \sum_{m=-\infty}^{\infty} x[m + l] y[m]
\end{align*}
\]

\[
x - \text{corr } y \text{ wrt } x \\
\begin{align*}
r_{yx}[l] &= \sum_{m=-\infty}^{\infty} y[m] x[m - l] \\
r_{yx}[l] &= \sum_{m=-\infty}^{\infty} y[m + l] x[m]
\end{align*}
\]

\[r_{xy}[l] = r_{yx}[-l] \ldots \text{Commutative?}\]
- $y[n]$ is said to be shifted by $l$ samples to the right with respect to the reference sequence $x[n]$ for positive values of $l$, and shifted by $l$ samples to the left for negative values of $l$.

- The ordering of the subscripts $xy$ in the definition of $r_{xy}[l]$ specifies that $x[n]$ is the reference sequence which remains fixed in time, while $y[n]$ is being shifted with respect to $x[n]$. 
Mathematically, Convolution between $x[n]$ & $h[n]$ is given as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \ h[n-k]$$

Correlation of $x[n]$ with $h[n]$ is given as

$$r[n] = \sum_{k=-\infty}^{\infty} x[k] \ h[k-n]$$

Only replaced variables $l$ by $n$ and $m$ by $k$.
But if we “time-reversed” the second sequence of the Convolution, we end up with Correlation

\[ y[n] = \sum_{k=-\infty}^{\infty} x[k] \ h[-(n-k)] \]

\[ = \sum_{k=-\infty}^{\infty} x[k] \ h[-n+k] \]

\[ r[n] = \sum_{k=-\infty}^{\infty} x[k] \ h[k-n] \]

Where, \( r[n] \) is the correlation of \( x[n] \) with respect to \( h[n] \)
So, we can say that

“Correlation, mathematically, is just Convolution, with the second sequence, time-reversed”

\[ r[n] = x[n] * h[-n] \]

We can use this property to find Correlation, using the same method we used for Convolution, albeit, the second sequence needs to be time-reversed

This only requires that we don’t time-reverse for convolution in the first place!
Example 1: Find the correlation b/w the two sequences \(x[n]\) and \(y[n]\) given by,

\[
x[n] = [3 \ 1 \ 2] \quad y[n] = [3 \ 2 \ 1]
\]

<table>
<thead>
<tr>
<th>(m)</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x[m]):</td>
<td></td>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y[m+2]):</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y[m+1]):</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y[m]):</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y[m-1]):</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y[m-2]):</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(y[m-3]):</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hint: The value of \(m\) starts from \((- \text{length of } y + 1)\) and continues till \((\text{length of } y + \text{length of } x - 1)\)

Here \(m\) starts from \(-3 + 1 = -2\) and continues till \(3 + 3 - 1 = 5\)
\[ r'_{xy[-2]} = 3 \times 1 = 3 \]
<table>
<thead>
<tr>
<th>m:</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x[m]:</td>
<td></td>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m+2]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m+1]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-1]:</td>
<td></td>
<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-2]:</td>
<td></td>
<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-3]:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    r_{xy[-2]} &= 3 \times 1 = 3 \\
    r_{xy[-1]} &= 3 \times 2 + 1 \times 1 = 7
\end{align*}
\]
<table>
<thead>
<tr>
<th>m:</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x[m]:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m+2]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m+1]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-1]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-2]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-3]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ r_{xy[-2]} = 3 \times 1 = 3 \]
\[ r_{xy[-1]} = 3 \times 2 + 1 \times 1 = 7 \]
\[ r_{xy[0]} = 3 \times 3 + 1 \times 2 + 2 \times 1 = 13 \]
<table>
<thead>
<tr>
<th>m</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>x[m]:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m+2]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m+1]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-1]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-2]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m-3]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    r_{xy[-2]} &= 3 \times 1 = 3 \\
    r_{xy[-1]} &= 3 \times 2 + 1 \times 1 = 7 \\
    r_{xy[0]} &= 3 \times 3 + 1 \times 2 + 2 \times 1 = 13 \\
    r_{xy[1]} &= 1 \times 3 + 2 \times 2 = 7
\end{align*}
\]
\[
\begin{array}{ccccccc}
m: & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
x[m]: & 3 & 1 & 2 & & & & & \\
y[m+2]: & 3 & 2 & 1 & & & & & \\
y[m+1]: & 3 & 2 & 1 & & & & & \\
y[m]: & 3 & 2 & 1 & & & & & \\
y[m-1]: & 3 & 2 & 1 & & & & & \\
y[m-2]: & 3 & 2 & 1 & & & & & \\
y[m-3]: & 3 & 2 & 1 & & & & & \\
\end{array}
\]

\[
\begin{align*}
r_{xy[-2]} &= 3 \times 1 = 3 \\
r_{xy[-1]} &= 3 \times 2 + 1 \times 1 = 7 \\
r_{xy[0]} &= 3 \times 3 + 1 \times 2 + 2 \times 1 = 13 \\
r_{xy[1]} &= 1 \times 3 + 2 \times 2 = 7 \\
r_{xy[2]} &= 2 \times 3 = 6
\end{align*}
\]
Define $x$ & $y$ in Matlab and use command `xcorr(x,y)` to verify this answer

<table>
<thead>
<tr>
<th>$m$:</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x[m]$:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y[m+2]$:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y[m+1]$:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y[m]$:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y[m-1]$:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y[m-2]$:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y[m-3]$:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
r_{xy}[-2] = 3 \times 1 = 3
\]
\[
r_{xy}[-1] = 3 \times 2 + 1 \times 1 = 7
\]
\[
r_{xy}[0] = 3 \times 3 + 1 \times 2 + 2 \times 1 = 13
\]
\[
r_{xy}[1] = 1 \times 3 + 2 \times 2 = 7
\]
\[
r_{xy}[2] = 2 \times 3 = 6
\]
\[
r_{xy}[3] = 0 \text{ (no overlap)}
\]

\[
r_{xy}(l) = \{3 \ 7 \ 13 \ 7 \ 6\}
\]
Example 2: Find the correlation of $y[n]$ with respect to $x[n]$, with the sequences given by,

$$x[n] = [3 \ 1 \ 2] \quad y[n] = [3 \ 2 \ 1]$$

<table>
<thead>
<tr>
<th>$m$</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y[m]$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x[m+2]$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x[m+1]$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x[m]$</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x[m-1]$</td>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x[m-2]$</td>
<td></td>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x[m-3]$</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hint: The value of $m$ starts from ($-\text{length of } y + 1$) and continues till ($\text{length of } y + \text{length of } x - 1$)

Here $m$ starts from $-3 + 1 = -2$ and continues till $3 + 3 - 1 = 5$
\[
\begin{array}{ccccccc}
m: & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\
y[m]: & 3 & 2 & 1 \\
x[m+2]: & 3 & 1 & 2 \\
x[m+1]: & 3 & 1 & 2 \\
x[m]: & 3 & 1 & 2 \\
x[m-1]: & 3 & 1 & 2 \\
x[m-2]: & 3 & 1 & 2 \\
x[m-3]: & 3 & 1 & 2 \\
\end{array}
\]

\[r'_{yx[-2]} = 3 \times 2 = 6\]
\[ r_{yx[-2]} = 3 \times 2 = 6 \]
\[ r_{yx[-1]} = 3 \times 1 + 2 \times 2 = 7 \]
### Table

<table>
<thead>
<tr>
<th>m</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y[m]</td>
<td></td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m+2]</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m+1]</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m]</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m-1]</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m-2]</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m-3]</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    r_{yx[-2]} &= 3 \times 2 = 6 \\
    r_{yx[-1]} &= 3 \times 1 + 2 \times 2 = 7 \\
    r_{yx[0]} &= 3 \times 3 + 2 \times 1 + 1 \times 2 = 13
\end{align*}
\]
<table>
<thead>
<tr>
<th></th>
<th>m:</th>
<th></th>
<th></th>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y[m]:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m+2]:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m+1]:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m]:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m-1]:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m-2]:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x[m-3]:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ r_{yx[-2]} = 3 \times 2 = 6 \]
\[ r_{yx[1]} = 2 \times 3 + 1 \times 1 = 7 \]
\[ r_{yx[-1]} = 3 \times 1 + 2 \times 2 = 7 \]
\[ r_{yx[0]} = 3 \times 3 + 2 \times 1 + 1 \times 2 = 13 \]
\[ r_{yx[-2]} = 3 \times 2 = 6 \]

\[ r_{yx[1]} = 2 \times 3 + 1 \times 1 = 7 \]

\[ r_{yx[-1]} = 3 \times 1 + 2 \times 2 = 7 \]

\[ r_{yx[2]} = 1 \times 3 = 3 \]

\[ r_{yx[0]} = 3 \times 3 + 2 \times 1 + 1 \times 2 = 13 \]
Define $x$ & $y$ in Matlab and use command `xcorr(y,x)` to verify this answer.

<table>
<thead>
<tr>
<th>$m$:</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y[m]$:</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$x[m+2]$:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$x[m+1]$:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$x[m]$:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$x[m-1]$:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$x[m-2]$:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$x[m-3]$:</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

\[
r_{yx[-2]} = 3 \times 2 = 6 \\
r_{yx[-1]} = 3 \times 1 + 2 \times 2 = 7 \\
r_{yx[0]} = 3 \times 3 + 2 \times 1 + 1 \times 2 = 13 \\
r_{yx[1]} = 2 \times 3 + 1 \times 1 = 7 \\
r_{yx[2]} = 1 \times 3 = 3 \\
r_{yx[3]} = 0 \text{ (no overlap)} \\
\]

\[
r_{yx}(l) = \{6, 7, 13, 7, 3\}
\]
Example 2: Find the correlation of the two sequences $x[n]$ and $y[n]$ represented by,

\[ x[n] = \{1 \ 2 \ 3\} \quad y[n] = [0.5 \ 1 \ 2 \ 1 \ 0.5] \]

Verify $R_{xy}[l] = R_{yx}[-l]$
Auto-Correlation

**Auto-Correlation:** Correlation between the same sequence

Mathematically, it is given as,

\[ r_{xx}[l] = \sum_{m=-\infty}^{\infty} x[m]x[m-l], \quad l = 0,\pm1,\pm2,\ldots \]

At zero(0) lag, it returns a maximum value and the energy of the signal

\[ r_{xx}[0] = \sum_{n=-\infty}^{\infty} x[m]x[m] = x^2[n] = E_x \]
Suppose, I have three sequences

\[ a[n] = [1 \ 2 \ 3 \ 4 \ 3 \ 2 \ 1]; \]
\[ b[n] = [4 \ 8 \ 12 \ 16 \ 12 \ 8 \ 4]; \]
\[ c[n] = [8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8]; \]

and I want to find the correlation between \( a[n] \) & \( b[n] \) and also \( a[n] \) & \( c[n] \)

i.e. \( r_{ab} \) & \( r_{ac} \)
The results show that

1) Correlation between $a[n] \& b[n]$ has a maximum value of 176 at zero lag

2) Correlation between $a[n] \& c[n]$ has a maximum value of 128 at zero lag

These results seem ambiguous and misleading, since $a[n]$ is a tri-pulse and $c[n]$ is simply a constant, while $b[n]$ is only a scaled version of $a[n]$. 
Normalized Correlations

- Normalized Correlation between $x[n]$ and $y[n]$

$$\rho_{xy}(l) = \frac{r_{xy}(l)}{\sqrt{r_{xx}(0)r_{yy}(0)}}$$

- Normalized Autocorrelation

$$\rho_{xx}(l) = \frac{r_{xx}(l)}{r_{xx}(0)}$$
Normalize the input sequences

Normalized sequence a

Normalized sequence b

Normalized sequence c

Correlation between a with respect to b

Correlation between a with respect to c
Normalize the input sequences and the correlation sequences.