Chapter 6

Capital and the marginalist theory

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6.1 Capital supply and demand (1)

**Equilibrium rate of interest:** “interest, being the price paid for the use of capital in any market, tends towards an equilibrium level such that the aggregate demand for capital in that market, at that rate of interest, is equal to the aggregate stock forthcoming there at that rate” (Marshall 1920, p. 534).
6.1 Capital supply and demand (2)

Supply of capital: stock of capital the households are endowed with.

According to Marshall (1920, p. 534), “it is only slowly and gradually that the rise in the rate of interest will increase the total stock of capital”, so that, for the purposes of the theory of value and distribution, capital accumulation could be neglected.

The supply of capital is a given magnitude: $K^S = \bar{K}$. 
According to **Wicksell** (1851-1926):

The real theoretical difficulty is [...] to explain how, under stationary conditions, the possession of capital can remain a permanent source of income. The application to non-stationary conditions offers no difficulty in principle. 

[...] Both logically and for purposes of exposition it would seem right to begin by examining the effects of a given supply of capital already accumulated, and then to inquire the causes which influence, and eventually alter, this supply. (*Lectures*, vol. I, pp. 154-5.)
6.1 Capital supply and demand (4)

**Demand for capital**: value of the capital goods employed.

The demand for capital is typically understood as a function of the price system, which includes the rate of interest: $K^D = K^D(\ldots, r)$.

Being a demand function, higher is the rate of interest, lower is the demand for capital. In other words, higher rates of interest bring about the use of less “capital-intensive” methods of production.
6.1 Capital supply and demand (5)

**Substitutability between labour and capital:** once capital and labour are placed on the same footing, they can be substituted for each other. One may think that there are different production methods for the same commodity that employ capital and labour in different proportions.

If the rate of interest increases in relation to the wage rate, firms are pushed to adopt methods that employ less capital and more labour.

Is this really the case?
6.2 Capital in Solow’s model (1)

In Solow’s model (1956), the existence of just one commodity—employed both for consumption and as a capital good—is assumed.

\[ Y = F(K,L) \quad \text{net output} \]
\[ Q = F(K,L) + K = G(K,L) \quad \text{gross output} \]

Hereafter we shall refer mainly to the net production function.

Let \( y = Y/L \) and \( k = K/L \) be the net output per worker and the employment of capital per worker respectively, because of the assumption of constant returns to scale: \( y = f(k) = F(k,1) \).
6.2 Capital in Solow’s model (2)

Let $w$ and $r$ be the wage rate and the rate of interest respectively.

Each firm decides its production plan—i.e. the employment of capital per unit of labour $k$—so to maximize its (extra)profits per unit of labour:

$$\pi = f(k) - w - rk$$

The first order condition is:

$$[1] \quad f'(k) - r = 0$$
6.2 Capital in Solow’s model (3)

Let us assume: $f'(k) > 0$ and $f''(k) < 0$.

Then, for a given $r$, the optimal employment of capital per unit of labour is determined by equation [1].

Therefore—as in the case with land and labour—the principle of diminishing marginal product implies that:

i) the function of demand for capital (per unit of labour) $k = k_d(r)$ derives from equation [1];

ii) this demand function has a decreasing trend: the higher the rate of interest $r$, the lower the demand for capital per unit of labour $k$. 
6.2 Capital in Solow’s model (4)

There is a unique and stable equilibrium:

\[ k_d(r^*) = k^* \]

in which \( k^* \) is the ratio between the available quantities of capital \( K^* \) and labour \( L^* \).

Once the equilibrium rate of interest \( r^* \) is determined, the wage rate is determined residually:

\[ w^* = f(k^*) - r^*k^* \]

and it is equal to the marginal product of labour.
6.3 Capital in Swan’s model (1)

Swan’s model (1956) is a two-sector model. Two commodities: a consumption good (corn) and a capital good (Meccano sets).

We denote by $Y$ the net product of the economy, which we assume consists of corn only.

The function $Y = F(M,L)$ represents the vertically-integrated production of corn.

We refer to quantities per unit of labour: $y = f(m) = F(m, 1)$, with $m = M/L$. 
Meccano sets
Meccano sets
BOX: Vertically-integrated sector (1)

Referring to the economy as a whole, the **net product** is the difference between the amount of commodities obtained as outputs and those employed as inputs.

\[
\text{Social Product – Means of Production} = \text{Social Net Product}
\]

Typically, the net product of a certain sector cannot be calculated. If we consider the process:

\[
A_a \oplus B_a \oplus \ldots \oplus K_a \oplus L_a \rightarrow A
\]

the output is a quantity \( A \) of commodity “\( a \)”, whereas the inputs include commodities “\( b \)”, “\( c \)”, …, “\( k \)” too.
BOX: Vertically-integrated sector (2)

However, it is possible to build a “vertically-integrated sector” whose net product is made of commodity “a”.

In the vertically-integrated sector of commodity “a”, the gross output includes a quantity $A$ of commodity “a” plus all the commodities employed as inputs in the sector itself:

$$A_{av} \oplus B_{av} \oplus ... \oplus K_{av} \oplus L_{av} \rightarrow (A + A_{av}) \oplus B_{av} \oplus ... \oplus K_{av}$$

For a vertically-integrated sector, the physical net output can be calculated and it is, in our example, a quantity $A$ of commodity “a”.
BOX: How to build a vertically-integrated sector (1)

Technical coefficients

0.6 units of meccano sets $\oplus$ 0.8 units of labour $\rightarrow$ 1 unit of corn

0.5 units of meccano sets $\oplus$ 1 unit of labour $\rightarrow$ 1 unit of meccano sets

$y$: net output per unit of labour in the v-i-s

$m$: employment of meccano sets per unit of labour in the v-i-s

The vertically integrated sector of corn employs a combination of inputs $(m,1)$ and its gross output is $(y,m)$

\[ 0.6y + 0.5m = m \]
\[ 0.8y + m = 1 \]

$y = 0.5$ and $m = 0.6$
6.3 Capital in Swan’s model (2)

Let \( p \) be the price of a Meccano set in terms of corn, each firm will choose its (vertically-integrated) method of production in order to maximize its (extra)profits:

\[
\pi = f(m) - w - r p m
\]

Accordingly, the first order condition is:

\[
[4] \quad f'(m) - r p = 0
\]

Equation [4] looks like equation [1], but the presence of the price \( p \) is an important difference.
6.3 Capital in Swan’s model (3)

In Swan’s model, we must distinguish between two different employments of ‘capital’:

i. the employment of capital goods, expressed by the employment of Meccano sets per unit of labour $m$;

ii. the employment of (value) capital, expressed by the amount of corn invested in Meccano sets per unit of labour $k = pm$.

The amount of capital goods $m$ enters into the production function, but the interest on capital is calculated on $k$. 
6.3 Capital in Swan’s model (4)

Can we get, from equation [4], an inverse relationship between $k$ and $r$?

By definition $k = pm$, therefore, differentiating by $r$ we get:

$$\frac{dk}{dr} = \frac{dp}{dr}m + p\frac{dm}{dr}$$

The variation of the rate of interest has a double effect on the value of capital per unit of labour:

• it makes the price of Meccano sets change ("price Wicksell effect");
• it involves a change in the employment of Meccano sets per unit of labour ("real Wicksell effect").
6.3 Capital in Swan’s model (5)

The sign of $dk/dr$ depends on how the price $p$ changes as $r$ varies.

Assuming that the employment of Meccano sets per unit of labour in the Meccano-set industry is higher than is the corn industry, then $p$ will rise as $r$ rises. Namely: $dp/dr > 0$. In this case, the price effect will be positive.

The real effect will have, on the contrary, negative sign. If $p$ rises as $r$ rises, then, because of equation [4] and the diminishing marginal productivity principle, $dm/dr < 0$.

There is not certainty about the sign of $dk/dr$. It depends on which of the two effects will prevail over the other.
BOX: Price effect (1)

The price of a Meccano set in terms of corn depends on the relative cost of the two commodities.

Let us denote by \( m_c \) and \( l_c \) the employments of Meccano sets and labour per unit of output in the corn sector. Similarly, \( m_m \) and \( l_m \) are the employments of Meccano sets and labour per unit of output in the Meccano-set sector.

Price equations:

\[
1 = p m_c (1 + r) + w l_c \\
p = p m_m (1 + r) + w l_m
\]
It follows that: \[ p = \frac{l_m}{l_c + (l_m m_c - l_c m_m)(1+r)} \]

Therefore, the variation of \( p \) as \( r \) rises depends on the ratio Meccano sets/labour in the two sectors. Three cases are possible:

- if \( m_c/l_c > m_m/l_m \), then \( p \) decreases as \( r \) rises;
- if \( m_c/l_c < m_m/l_m \), then \( p \) rises as \( r \) rises;
- if \( m_c/l_c = m_m/l_m \), then \( p \) does not change as \( r \) rises.
6.3 Capital in Swan’s model (6)

Two further remarks:

**A)** Given the first order condition \( f'(m) = pr \), if \( p \) rises as \( r \) rises, then it is evident that \( m \) decreases as \( r \) increases. However, albeit less evident, this result also holds when \( p \) diminishes as \( r \) rises (Cf. Fratini 2013).

**B)** If \( dp/dr = 0 \), then (because of equation [5]) \( dk/dr = p \cdot dm/dr \). Therefore, the value capital per unit of labour always changes in the same direction as the employment of Meccano sets per unit of labour and, accordingly, it falls as \( r \) increases.

But in this case we are back to Solow’s model because \( m_c/l_c = m_m/l_m \) means that corn and Meccano sets are actually the same commodity.
Let us assume that the entire net product is adsorbed by the payment of wages and interests (zero extra-profit):

\[ y = w + rk \]

Differentiating equation [6] with respect to \( r \), we get:

\[ \frac{dy}{dr} = \frac{dw}{dr} + k + r \frac{dk}{dr} \]

Re-organizing the terms in equation [7] and dividing by \( dk/dr \), we get:

\[ \frac{dy/dr}{dk/dr} = r + \frac{1}{dk/dr} \left( \frac{dw}{dr} + k \right) \]
6.4 Wicksell effect (2)

Interpreting \((dy/dr)/(dk/dr)\) as the ‘marginal product of capital’ (in value terms), such a marginal product can be equal to \(r\) if and only if:

\[
[9] \quad dw/dr + k = 0
\]

Since, generally, \(dw/dr + k \neq 0\), then the marginal product of capital is not equal to the rate of interest.

This result was already observed by Wicksell in his *Lectures* (1934 [1911]) and was, therefore, called “Wicksell effect”.

6.4 Wicksell effect (3)

In order to verify that, in general, \( dw/dr + k \neq 0 \), let us go back to Swan’s model.

Bearing in mind that \( k = pm \), in Swan’s model equation [6] becomes:

\[
10 \quad y = w + rpm
\]

Differentiating equation [10] with respect to \( r \), we get:

\[
11 \quad \frac{dy}{dr} = \frac{dw}{dr} + mp + rp \frac{dm}{dr} + rm \frac{dp}{dr}
\]
6.4 Wicksell effect (4)

The first order condition $f'(m) = rp$ implies that:

$$\frac{dy}{dm} = rp\quad \text{or} \quad \frac{dy}{dr} = rp\frac{dm}{dr}$$

then, substituting equation [12] into equation [11], we get:

$$\frac{dy}{dr} = \frac{dw}{dr} + mp + \frac{dy}{dr} + rm\frac{dp}{dr} \quad \text{or} \quad \frac{dw}{dr} + mp = -rm\frac{dp}{dr}$$

Therefore, assuming that $r$ and $m$ are both strictly positive, then we can have $dw/dr + mp = 0$ if and only if $dp/dr = 0$, namely the price of Meccano sets in terms of corn does not depend on the rate of interest (but this implies that Meccano sets and corn are the same commodity).
6.5 Many capital goods (1)

Let us consider an economy with \( M \) different commodities.

Commodity 1 is used both for consumption and as a capital good (as corn in Solow’s model). Commodities 2, 3, ..., \( M \) are pure (circulating) capital goods (as Meccano sets in Swan’s model).

Let us denote by \( Y \) the net output of the vertically integrated sector of commodity 1.

\( K_1, K_2, \ldots, K_M \) and \( L \) are the quantities of capital goods and labour employed as inputs in the vertically integrated sector of commodity 1.


6.5 Many capital goods (2)

The net output depends on the quantities of the inputs employed:

\[ Y = F(K_1, K_2, \ldots, K_M, L) \]

Can we get economically meaningful marginal products from the partial derivatives of the above function?

Capital goods are highly *specialized* inputs: they are invented to be used in a specific way. Usually, the invention of new kinds of capital goods corresponds to the invention a new methods of production.

The partial derivatives of the function \( Y = F(K_1, K_2, \ldots, K_M, L) \) either do not exists or are nil.
6.5 Many capital goods (3)

On the one hand, if a change of the methods of production in use occurs, it implies a change of the kinds of capital goods employed, and not merely a variation in the quantities of the same kinds of capital goods.

On the other hand, if the methods are not changed, then the inputs employed are complementary among themselves, so that there is no increase in the output if the employment of just one input is increased with given the quantities of all the others.

In both cases, a meaningful (non-zero) marginal product of a specific kind of capital goods does not obtain.
6.5 Many capital goods (4)

**Conclusion**

In order to have meaningful marginal products, all the capital goods should consist of an ultimate substance: “aggregate capital”.

Since the birth of the marginalist theory, many attempts have been made: no one has worked.

In particular: one cannot consider the value of the capital goods employed as the quantity of a factor of production.
6.6 Criticism of the neoclassical parable (1)

**First proposition**: Before a level of the rate interest is determined, it is not possible to say that one method of production is more capital-intensive than another (cf. Sraffa 1960, p. 38).

Let us assume there are 2 methods of production of the same commodity. Let \( k_I \) and \( k_{II} \) be the values of the capital goods employed (per unit of labour) with methods \( I \) and \( II \) respectively. The ratio \( k_I/k_{II} \) varies with the rate of interest and:

- \( k_I/k_{II} > 1 \) if \( r \in S \)
- \( k_I/k_{II} \leq 1 \) if \( r \in S' \)
6.6 Criticism of the neoclassical parable (2)

Second proposition: if an increase of the rate of interest entails a change of the method of production in use, then the incoming method is not necessarily less capital-intensive then the outgoing and does not necessarily give a smaller output per unit of labour (Sraffa 1960, ch. XII).

Re-switching is possible. Let us consider two methods I and II, with $y_{II} > y_I$. Let us imagine increasing the interest rate between 0 and $R$. For $r$ close to 0, method II is in use. After a certain level $r' > 0$, method I becomes the mothed in use, but after $r'' > r'$ method II can be in use again.
6.6 Criticism of the neoclassical parable (2)

Sraffa 1960, p. 81
6.7 Conclusions (1)

1. Capital is not a factor of production. It is not on the same footing as labour. In general, we cannot say that a method is more capital-intensive than another. This is not a technical property.

There often turns out to be no unambiguous way of characterizing different processes as more ‘capital-intensive,’ more ‘mechanized,’ more ‘roundabout,’ except in the ex post tautological sense of being adopted at a lower interest rate (Samuelson, 1966, p. 582)
6.7 Conclusions (2)

2. The neoclassical parable does not work. A fall in the rate of interest does not lead necessarily to an increase in the employment of capital per unit of labour and in the output per unit of labour.

The simple tale told by Jevons, Böhm-Bawerk, Wicksell, and other neoclassical writers—alleging that, as the interest rate falls in consequence of abstention from present consumption in favor of future, technology must become in some sense more ‘roundabout,’ more ‘mechanized,’ and ‘more productive’—cannot be universally valid. (Samuelson, 1966, p. 582)
6.7 Conclusions (3)

3. The rate of interest in not the price for the use of capital.

The value which accrues from a sale is the product of price and quantity sold. Hence if the rate of interest is the price of capital, the quantity of capital must be the wealth on which an interest yield is calculated. It will be shown shortly why this view is incorrect, but to cut a long story short, the conclusion may be announced at once. The rate of interest is not the price of capital. (Bliss 1975, pp. 6-7)