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Information Quality and Stock Returns Revisited

Frode Brevik and Stefano d'Addona*

Abstract

This paper investigates the relation between information on the state of the economy and equity risk premium. We use a setup where investors have Epstein-Zin preferences and the economy randomly switches between booms and recessions. We are able to establish 2 key results: First, investors with high elasticity of intertemporal substitution (EIS) will require lower excess returns for holding stocks if they are provided with better information on the state of the economy. Second, we find that this also holds for investors with moderate EIS if they are sufficiently risk averse.

I. Introduction

Publicly available signals might contain more or less information on the underlying state of the economy. High-quality signals will enable investors to make high-quality forecasts on the state of the economy, so it is natural to expect risk premiums to vary with the amount of information signals contain.

In an important contribution, Veronesi (2000) studies the link between information quality and risk premiums in an exchange economy where the trend growth rate follows a hidden Markov process. Assuming that the representative investor is a power utility maximizer, he establishes several surprising relations between the information on the state of the economy and stock returns, including the following: i) If the representative investor has a relative risk aversion (RA) larger than unity, then the risk premium is *increasing* in the amount of information contained in the signals, and ii) unless the signals contain complete information

*Brevik, fbrevik@feweb.vu.nl, VU University Amsterdam, De Boelelaan 1105, 1081 Amsterdam, Netherlands; d'Addona, daddona@uniroma3.it, University of Rome 3, Via G. Chiabrera, 199, I-00145 Rome, Italy, and Baruch College–City University of New York, One Bernard Baruch Way, New York, NY 10010. We are grateful to Sergio Albeverio, Itamar Drechsler, Carlo Savino, Pascal St-Amour, Marco Taboga, Pietro Veronesi, Kamil Yilmaz, and Zhaoxia Xu, and the seminar participants at the University of Bonn, the University of St. Gallen, the University of Rome 3, the 2006 Canadian Economics Association (CEA) meeting, the 2006 European Financial Management Association (EFMA) meeting, the 2006 Western Economic Association International (WEAI) meeting, and the 2007 Financial Management Association (FMA) meeting for their comments. We especially thank Stephen Brown (the editor) and Anthony Lynch (the referee), whose comments have significantly improved this paper.

on the state of the economy, the equity premium is *bounded above* independently of investor RA. The second result implies that, even with extremely risk-averse investors, the model would not be capable of replicating the empirically observed risk premium.

The first result has been reviewed in different setups by Li (2005) and Ai (2005). Li introduces a separate process for dividends and looks at 2 special cases. When the conditional mean growth rate of consumption and dividends is the same, he obtains the same results as Veronesi (2000): When only the dividend growth rate is time varying, better information lowers the equity premium. Ai looks at a production economy where equity is a claim to 1 unit of capital and there are no adjustment costs. He finds a negative relation between information quality and the required return to capital. Since the price of capital in terms of the consumption good is constant in his model, it is not clear how his results would translate to the exchange economy setting investigated by Veronesi (2000).

In this paper, we maintain both the pure exchange economy setting and the assumption that equity is a claim to aggregate consumption. We revisit the relation between information quality and stock returns by introducing the Epstein-Zin (1989) preferences to Veronesi's (2000) model. Because the Epstein-Zin (1989) preferences nest the power utility function as a special case, we are able to build on Veronesi's (2000) work and provide a direct comparison with his results. As we show numerically, both results are reversed for a range of plausible parameters in an economy calibrated to U.S. data. Although a large literature explores the asset-pricing implications of alternative preference specifications, we are not aware of anyone who specifically addresses the topic of information quality.[1]

The main finding of our paper is that, for a wide range of plausible parameterizations of the utility function in an economy calibrated to U.S. data, the conditional equity premium is decreasing in the quality of information available to investors. This range covers both a domain where this reversal has been predicted in the literature, which is when the elasticity of intertemporal substitution (EIS) is greater than 1, as well as an important domain where the EIS is smaller than 1, provided that investors are sufficiently risk averse. Both results are important, since there is considerable controversy with respect to the appropriate parameter value for the EIS.[2]

[1]One particularly prominent line of research looks at the asset-pricing implications of including habits in the utility function (e.g., Constantinides (1990), Abel (1990), Gali (1994), Jermann (1998), Campbell and Cochrane (1999), and Boldrin, Christiano, and Fisher (2001)). Another line of research, started by Epstein and Zin (1989), (1991) and Weil (1989), looks at generalizations of the power utility function that relax the link between RA and EIS. This class of utility functions, referred to as the Epstein-Zin (1989) preferences, has been widely used in recent research in asset pricing (see Campbell and Viceira (2001), Campbell, Chan, and Viceira (2003), Guvenen (2006), and many others), including some featuring the same kind of Bayesian learning we are assuming (Brandt, Zeng, and Zhang (2004), Lettau, Ludvigson, and Wachter (2008)).

[2]Empirical estimates vary strongly with the assumptions made on the structure of the economy. One line of empirical research uses a representative agent setup and estimates the EIS parameter using aggregate consumption data. This approach typically leads to estimated EIS coefficients in the range of 0 to 1 (see, e.g., Hall (1988), Campbell and Mankiw (1989), (1991), Hahm (1998), Yogo (2004), and Zhang (2006)). Another line of research seeks to avoid potential biases, introduced by using aggregate data, by relying on microeconomic survey data. For stockholders, these studies find EIS parameters around or above 1 (see Beaudry and van Wincoop (1996), Vissing-Jørgensen (2002),

An alternative to introducing hidden growth rate regimes is to model consumption growth rates as having a slow-moving predictable component. This is the approach taken by Bansal and Yaron (2004). Although they also use the Epstein-Zin (1989) preferences, their results are different from ours. They report that if investors have a preference for early resolution of uncertainty, an EIS larger than 1 is required for the equity premium to be increasing with uncertainty. The reason for the discrepancy with our results lies in a different understanding of uncertainty. In Bansal and Yaron's model, the state of the economy is observable, and uncertainty is understood as conditional consumption volatility. In our model, uncertainty is lack of knowledge about the prevailing state of the economy.

Another finding is that in our setup there is no global maximum for the required equity premium as a function of investor RA. This is different from what obtains in a regime-switching economy, where investors are power utility maximizers (see Prop. 3b in Veronesi (2000)). Unless the EIS parameter is very low, increasing investor RA in our model leads monotonically to a higher required equity premium, at least for the economies we consider numerically.

While we allow for a more general utility function than Veronesi (2000), we remain close to his model in terms of the dynamics of the model economy. We assume that the trend growth rate follows a 2-state Markov switching process. The current trend growth rate is a hidden variable, so investors have to rely on the information embedded in dividend growth rates and other signals for pricing equities and bonds.[3]

This paper is organized as follows. Section II introduces the general model and the properties of the external signal, Section III shows the estimation of the process parameters for the U.S. economy, and Section IV presents pricing formulas, some qualitative results, and some conjectures for some special cases. Quantitative results for a wide range of preference parameters are provided in Section V. Section VI presents our conclusions. Proofs, algebraic derivations, and additional results are provided in the Appendices.

II. Model

We assume a pure exchange economy as in Lucas (1978). The economy is populated by a continuum of identical agents with the Epstein-Zin (1989) preferences given by

$$(1) \qquad V_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\mathcal{R}_t(V_{t+1})^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}.$$

Vissing-Jørgensen and Attanasio (2003), and Guvenen (2006)). Recent literature on asset pricing relies on the higher EIS estimates of the previously cited literature (e.g., Bansal and Yaron (2004) and Lettau et al. (2008) both calibrate their models with an EIS greater than 1).

[3]Because hidden Markov models of this class are able to capture regularities found in the data that are missed by linear models (see the discussion in Hamilton (2005)), they have been widely used in economics since Hamilton (1989). In particular, in the asset-pricing literature, the implications of a Markov switching process in the conditional mean of the endowment process are analyzed by Cecchetti, Lam, and Mark (1990), (1993), Kandel and Stambaugh (1991), Abel (1994), (1999), Veronesi (1999), Whitelaw (2000), Lettau et al. (2008), and many others.

The operator $\mathcal{R}_t$ makes a risk adjustment to the date $t+1$ continuation value. The risk adjustment is given by

$$\mathcal{R}_t(V_{t+1}) = \left(\mathrm{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}.$$

The parameter γ is the coefficient of RA, while the EIS is given by ψ. The function reduces to a monotone transformation of the standard power utility function for $\psi = \gamma^{-1}$. Dividends (the endowment good) grow according to the process

$$C_t = C_{t-1}e^{\mu_i + \sigma_c \epsilon_t}, \tag{2}$$

where μ_i denotes the mean log consumption growth rate in state i, σ_c is its state-invariant standard deviation, and ϵ_t is an independent and identically distributed (i.i.d.) standard normal noise term. The underlying state of the economy s_t follows an ergodic 2-state Markov chain with transition probability matrix between time t and $t+1$ given by

$$\Theta = \begin{pmatrix} \theta_1 & (1-\theta_2) \\ (1-\theta_1) & \theta_2 \end{pmatrix}. \tag{3}$$

The element (i,j) of the matrix denotes $\Pr(s_{t+1} = i \mid s_t = j)$. In general, we will assume that $\theta_i > 0.5$. For identification, we assume $\mu_1 > \mu_2$, so that the 1st state has the natural interpretation of a boom state, while the 2nd state is a recession state.

The state of the economy is not directly observable, but agents have various sources of information at hand for inferring it. The most obvious of these sources is the growth rate of the dividends themselves. Given the structure of the economy, which is assumed to be known to the agents, high growth rates indicate a high probability of being in the boom state, whereas the reverse is true for low growth rates.

All information in addition to that contained in dividend growth rates is aggregated as an independent signal. For convenience, we let the signal in state i take the form

$$y_t = \mu_i + \sigma_y \nu_t, \tag{4}$$

where ν_t is an i.i.d. standard normal noise term. The precision of the external signal is typically defined as $1/\sigma_y$. As σ_y goes to 0, the precision of the external signal goes to infinity, and as σ_y grows, the precision goes to 0. Instead of working directly with the precision of the signal, we prefer working with the variable $h \in [0, 1]$, which we call the strength of the signal. The strength of the signal is defined as the percentage reduction of the probability of receiving a signal in state i that has a higher likelihood in state j. Working with the strength of the signal has the advantage of being easier to interpret: When $h = 0$, the signal contains no information, whereas when $h = 1$ the signal is strong enough to reveal the state of the economy with certainty.

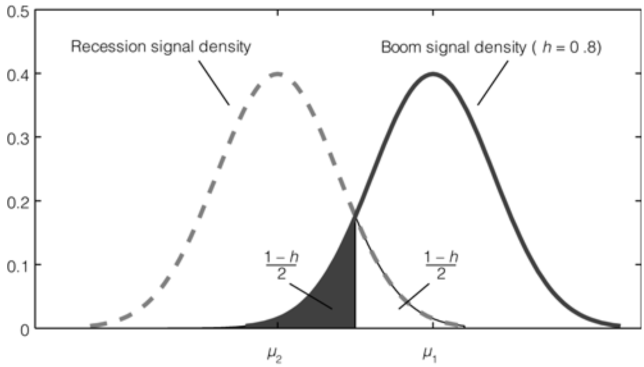
In the case where the external signal is pure noise, the probability of assigning a lower posterior probability to the true state based on only one realization of the signal (i.e., the probability of making a type I error) is 50%. For intermediate levels of signal strength, we let h be the percentage reduction in this probability relative to the pure noise case.[4]

Figure 1 illustrates that, based on a single observation, a higher probability is assigned to the state where the density is highest for that observation. Thus, the probability of a type I error is given by the area of the shaded region. It converges to 0.5 as the means of the 2 distributions converge.

FIGURE 1

Signal Strength and Signal Densities

Figure 1 plots the densities of the signal in the 2 states when the signal has strength $h = 0.8$. The area of the dark shaded region, which is $(1 - h)/2$, gives the probability of assigning a higher likelihood to the recession state, based on a single observation when the true state of the economy is a boom. Conversely, the area of the light shaded region gives the probability of assigning a higher likelihood of being in a boom when the true state of the economy is a recession.



III. Data and Estimation

The sample period chosen for calibrating the model spans the first quarter of 1952 to the last quarter of 2006. Prices and dividends are on the Standard & Poor's (S&P) 500 composite, while the risk-free rate is the yield on 1-year T-bills. These series were taken from Robert J. Shiller's Web site.[5] Consumption is quarterly real total personal consumption expenditures (National Income and Product Accounts, Table 2.3.6, line 1), as stated on the Bureau of Economic Analysis (BEA) Web site.[6] Finally, we use the official recession dates as reported on the Web site of the National Bureau of Economic Research (NBER).[7]

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[4]There is a 1-to-1 relation between the strength of the signal (h) and the precision of the signal ($1/\sigma_y$). This is implicitly given by

$$\sigma_y = -\frac{\mu_1 - \mu_2}{2F^{-1}\left(\frac{1-h}{2}\right)},$$

where F denotes the cumulative distribution function for a standard normal.

[5]http://www.econ.yale.edu/~shiller/data.htm.

[6]http://www.bea.gov/.

[7]http://www.nber.org/cycles/recessions.html.

Our data set is a standard data set, and our descriptive statistics are similar to those reported elsewhere in the literature. The average return on equity is 12.2% on an annual basis with a standard deviation of 11.7%. Compared with the mean risk-free rate of 2.8%, this yields an equity premium of around 9%. These numbers are summarized in Table 1.

TABLE 1

Descriptive Statistics

Table 1 summarizes annualized means and standard deviations in percentage points for key U.S. time series. Equity return is the real return from holding the S&P 500 composite, the risk-free rate is the real yield on 1-year T-bills, and consumption is real per capita personal consumption expenditures (Q1:1952–Q4:2006; sources: BEA and Robert Shiller's Web site).

	Equity Return	Risk-Free Rate	Consumption Growth
Mean	12.2	2.8	2.3
Std. deviation	11.7	1.1	1.4

Financial market lore contends that prices move procyclically with the business cycle. To verify this conjecture, we calculate the correlation matrix among the cyclical components of the U.S. economic and financial series. For this, all series were expressed in real terms, logged, and then filtered with the Hodrick and Prescott (HP) (1997) filter. As shown in Table 2, the cyclical components of all the series are strongly positively correlated, with a correlation coefficient ranging from 0.26 for the gross domestic product (GDP) and the price-dividend (PD) ratio to 0.94 for stock prices and the PD ratio.

TABLE 2

Cyclical Correlations

Table 2 reports the correlation matrix for the cyclical component of some U.S. financial and economic series, as obtained by detrending them with the Hodrick-Prescott (HP) filter. The S&P 500 is the level of the S&P 500 composite, the PD ratio is the price-dividend ratio for the S&P 500 composite, the GDP is the real per capita gross domestic product, and consumption is real per capita personal consumption expenditures (Q1:1952–Q4:2006; sources: BEA and Robert Shiller's Web page).

	S&P 500	PD Ratio	GDP	Consumption
S&P 500	1.000			
PD ratio	0.938	1.000		
GDP	0.370	0.257	1.000	
Consumption	0.439	0.345	0.878	1.000

Parameter estimates for the regime-switching model for the consumption series were found by a Markov chain Monte Carlo (MCMC) algorithm similar to that described in Section 9.1 of Kim and Nelson (1999). The resulting estimates are given in Table 3.

The probabilities of switching from the 2 states are 6.30% and 23.08%, respectively. These probabilities imply an average duration of 15.9 quarters for booms and 4.3 quarters for recessions.

Figure 2 shows that the Markov switching model is able to capture fairly well U.S. recessions as chronicled in the official NBER business cycle reference dates: The gray areas indicate the official recession periods, while the solid line gives the smoothed probabilities of being in a recession state.

TABLE 3

Estimated Model Parameters

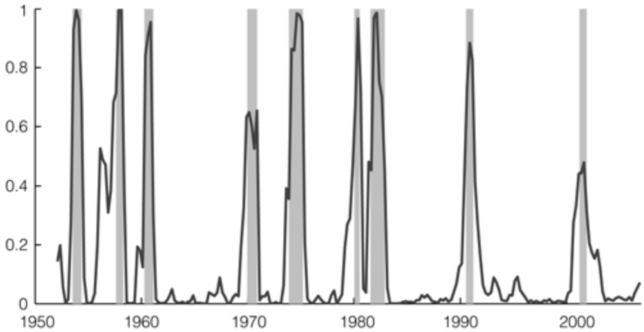
The estimated model economy parameters reported in Table 3 are based on an MCMC algorithm from Kim and Nelson (1999) using real quarterly per capita personal consumption expenditures (Q1:1952–Q4:2006; source: BEA). Standard errors are reported in parentheses.

	μ_i	σ_C	$(1-\theta_i)$
Boom	0.0074 (0.0005)	 0.0061	0.0630 (0.0255)
Recession	–0.0009 (0.0010)	(0.0003)	0.2308 (0.0780)

FIGURE 2

Model Implied Recession Probabilities

Figure 2 plots the smoothed recession probabilities, computed by applying the Hamilton (1989) filter to the U.S. consumption series, coupled with the official NBER recession dates (shaded areas).



IV. Log-Linear Results and Conjectures

One of the key results of Epstein and Zin (1989) is that the stochastic discount factor for the recursive utility function in equation (1) can be expressed as

$$(5) \qquad M_{t+1} \;=\; \beta^{\kappa}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\kappa}{\psi}}(R^e_{t+1})^{\kappa-1},$$

where $\kappa \equiv (1-\gamma)/(1-1/\psi)$ and R^e_{t+1} is the equilibrium gross return to aggregate wealth between t and $t+1$. Denoting the price-consumption ratio at time t by W_t, R^e_{t+1} can be expressed as $(C_{t+1}+C_{t+1}W_{t+1})/(C_tW_t)$.

Using equation (5), we can find expressions for the equity premium, as well as the 1-period real risk-free rate. As usual, the gross risk-free rate is given by the inverse of the expected value of the stochastic discount factor, or

$$(6) \qquad R^f_t \;=\; \mathrm{E}_t\left[\beta^{\kappa}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{1+W_{t+1}}{W_t}\right)^{\kappa-1}\right]^{-1}.$$

Thus, in this setting, the real interest rate will fluctuate not only with the expected growth rate of consumption but also with the expected changes in the price-consumption ratio.

We use the Euler equation for the claim to aggregate consumption to derive its price. Substituting for M_{t+1} and R^e_{t+1} in

$$1 = \mathrm{E}_t\left[M_{t+1}R^e_{t+1}\right] \tag{7}$$

and multiplying both sides by W_t^κ gives

$$W_t^\kappa = \mathrm{E}_t\left[\beta^\kappa\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}(1+W_{t+1})^\kappa\right]. \tag{8}$$

We follow Lettau et al. (2008) and solve this equation numerically, using the property that the boom-state probability is a sufficient statistic for the investors' time t information set.[8]

Since we are in an endowment economy, equilibrium requires that consumption always be equal to the dividends of the claim to the aggregate endowment, so the PD ratio is also given by equation (8). We will use the 2 terms interchangeably.

For the discussion in the next section, one key concern is to determine whether the prices predicted by the model are moving procyclically or countercyclically with the (perceived) state of the economy. We restrict our attention to the relevant case where the probability of the economy remaining in the same state, on a period-by-period basis, is higher than the probability of a regime switch. Appendix B establishes the following proposition:

Proposition 1. For $\theta_1, \theta_2 > ½$ if $\psi > 1$ ($\psi < 1$), the PD ratio is increasing (decreasing) in investors' posterior boom probability. If $\psi = 1$, the PD ratio is constant and equal to $1/(1-\beta)$.

Proof. See Appendix B. □

A. Expected Returns

We will now provide an intuition for Proposition 1 by showing that the expected growth rate of dividends influences not only expected future cash flows but also the rate at which they are discounted. Log-linearizing the Euler equation for equity and solving it for expected returns gives us a simplified framework to analyze it. The conditional expected log returns to equity ($\mathrm{E}_t\left[r^e_{t+1}\right]$) and the risk-free rate (r^f_t) can be expressed as

$$\begin{aligned}\mathrm{E}_t\left[r^e_{t+1}\right] &= -\log\beta + \frac{1}{\psi}\mathrm{E}_t\left[g_{t+1}\right] \\ &\quad - \frac{\kappa}{2}\left[\frac{1}{\psi^2}\sigma_t^2(g_{t+1}) + \sigma_t^2(r^e_{t+1}) - 2\frac{1}{\psi}\mathrm{cov}_t\left(g_{t+1}, r^e_{t+1}\right)\right],\end{aligned} \tag{9}$$

$$r^f_t = -\log\beta + \frac{1}{\psi}\mathrm{E}_t\left[g_{t+1}\right] - \frac{\kappa}{2}\left[\frac{1}{\psi^2}\sigma_t^2(g_{t+1}) + \left(\frac{1}{\kappa}-1\right)\sigma_t^2(r^e_{t+1})\right], \tag{10}$$

[8]The details of the numerical procedure are provided in Appendix A.

where $\sigma_t^2(g_{t+1})$ and $\sigma_t^2(r_{t+1}^e)$ denote the conditional variance of the log consumption growth rate and the log return to equity, respectively. Their conditional covariance is denoted by $\text{cov}_t\left(g_{t+1}, r_{t+1}^e\right)$.

The main difference between the 2 states is that the expected conditional growth rate of dividends is higher in booms than in recessions, so that the 2nd term of equation (9) will be higher. On the one hand, an upward revision of the conditional expected growth rate of dividends increases the expected payoffs of equity, increasing its value to investors. On the other hand, investors prefer consumption profiles that are smooth over time. Given an upward revision in the conditional expected dividend growth rate, investors would like to smooth their intertemporal consumption profile by shifting consumption from the future to the present. Since the model does not allow for any aggregate saving or dissaving, equilibrium can only be obtained if the conditional expected returns on all assets increase sufficiently to check the investors' desire to sell them off in order to finance consumption increases. The amount by which conditional expected returns will have to increase to maintain equilibrium depends on how tolerant investors are to consumption variations over time (i.e., on their EIS). If $\psi < 1$, an upward adjustment of the conditional expected dividend growth rate causes an even larger upward adjustment of the required return to equity. This leads to a drop in prices. If $\psi > 1$, an upward adjustment of conditional expected consumption growth rates is matched by a less than 1-to-1 adjustment of the required return to equity; hence prices would be increasing in the boom probability. By Proposition 1, we know that this result from the log-linear approximation holds generally.

B. Equity Returns

Having clarified how the conditional expected log return is determined in our setup, we turn our focus to the relation we want to analyze: the influence of information quality on the conditional equity premium. The following proposition provides an approximate analytical expression for assessing it.[9]

Proposition 2. If consumption growth and asset returns are conditionally jointly log normal, the conditional equity premium is given by

$$\text{(11)} \quad \begin{aligned} \mathrm{E}_t\left[r_{t+1}^e\right] - r_t^f + \frac{1}{2}\sigma_t^2(r_{t+1}^e) &= \gamma\sigma_t^2(g_{t+1}) + (1-\kappa)\,\sigma_t^2(\omega_{t+1}) \\ &\quad + ((1-\kappa)+\gamma)\,\text{cov}_t\left(g_{t+1}, \omega_{t+1}\right), \end{aligned}$$

where $\sigma_t(g_{t+1})$ is the conditional standard deviation of the log consumption growth, $\sigma_t^2(\omega_{t+1})$ is the conditional variance of $\log\left((1+W_{t+1})/W_t\right)$, and $\text{cov}_t\left(g_{t+1}, \omega_{t+1}\right)$ is their conditional covariance.

Proof. See Appendix B. □

In the following, we analyze the terms in equation (11) individually.

[9] The quantitative results in the next section do not rely on this linear approximation but, rather, on precise numerical algorithms.

C. Information and Equity Premium

The 1st part of equation (11), $\gamma\sigma_t^2(g_{t+1})$, is the familiar textbook formula for the conditional equity premium. Increasing the quality of the signal decreases the conditional volatility of consumption independently of RA and the EIS. Information quality also affects returns through their conditional variance and their conditional covariance with consumption. We will look at each of the two in turn.

1. Contribution of Returns Variance

We now conjecture that if $\psi \neq 1$, the conditional variance of equity returns net of dividend growth ($\log[(1+W_{t+1})/W_t]$) is increasing in signal quality.[10] One way to think about this conjecture is along the lines of Shiller (1981), who shows that the perfect foresight price of a stock will be more variable than the price with a smaller information set because expectations are smoother than realizations. The analogy to Shiller's argument does not carry over perfectly to our model, because the PD ratio is nonlinear and because we are analyzing returns instead of prices directly. Instead we can appeal to the following reasoning.

From equation (8) we know that the current PD ratio is uniquely determined by the posterior beliefs that investors assign to each of the 2 states. In the extreme case where the signal is strong enough to reveal the state of the economy with certainty (i.e., $h = 1$), the PD ratio will be constant as long as the underlying state of the economy does not change. Whenever the state of the economy switches, this will be detected immediately, and the PD ratio will jump straight to its new value. This implies a relatively large deviation of $\log[(1 + W_{t+1})/W_t]$ from its conditional mean.

The noisier the external signal, the harder it will be to detect regime changes. Learning about the new state of the economy will be protracted, resulting in smaller deviations of $\log[(1 + W_{t+1})/W_t]$ from its conditional mean as W moves toward the value predicted by the new state of the economy. Because the variance of a random variable is the expectation of the *squared* deviations of the variable from its mean, this is likely to lead to a lower variance.

If the conditional variance of returns net of dividend growth increases with the quality of the external signal, the effect of information quality on the conditional equity premium through this channel depends on the coefficient $(1 - \kappa)$. The following conjecture immediately follows.

Conjecture 1. We conjecture that the contribution of better information to the conditional equity premium through the conditional variance of returns is

- negative if $\kappa > 1$,
- 0 if $\kappa = 1$,
- positive if $\kappa < 1$.

[10] In Section V we show that the conjecture holds numerically for all the parameterizations we analyze with $\psi \neq 0$. The returns-based Euler equation we use in the log-linearization is not defined at $\psi = 1$.

Corollary 1
i) If investors have a preference for early resolution of uncertainty ($\gamma > 1/\psi$) and a coefficient of relative RA greater than unity, then the contribution of better information through the conditional variance of returns is

- negative if $\psi < 1$,
- positive if $\psi > 1$.

ii) In the power utility case the contribution of better information through the conditional variance of returns is 0.

2. Contribution of the Covariance of Returns and Consumption

In addition to the effect of the conditional variance of returns net of dividend growth, equation (11) shows that the conditional covariance with consumption growth rates will also have an effect on the equilibrium equity premium. It follows from Proposition 1 that the conditional covariance will be positive whenever ψ is greater than 1 and negative otherwise.

We conjecture that a better external signal reduces (in absolute terms) the conditional covariance of returns net of dividend growth and the consumption growth rate. By equation (8), the PD ratio is fully determined by the probabilities that investors assign to the 2 growth states. The more informative the external signal is, the less weight investors give to consumption growth rates when updating their beliefs on the state of the economy. Thus, the conditional covariance of prices with consumption growth rates decreases (in absolute terms) with information quality.

The impact of information quality on the required risk premium will depend on the sign of the conditional covariance and on the sign of its coefficient. The conditional covariance is positive if returns net of dividend growth are procyclical (when $\psi > 1$) and negative if returns net of dividend growth are countercyclical (when $\psi < 1$). Solving for the sign of the coefficient $(1 - \kappa + \gamma)$, this implies the following:

Conjecture 2
i) If investors have a preference for early resolution of uncertainty ($\gamma > 1/\psi$) and an RA parameter larger than unity ($\gamma > 1$), we conjecture that the contribution of better information quality on the conditional equity premium through the conditional covariance of returns net of growth and consumption growth rates is

- positive if $\psi \leq ½$,
- negative if $½ < \psi < 1$ and $\gamma > 1/(2\psi - 1)$,
- negative if $\psi > 1$.

ii) In the power utility case, the contribution is negative if $\psi > 1$.

An implication of this intuition-based conjecture is that if returns are procyclical (EIS > 1), the condition that investors have an RA parameter above unity is sufficient to ensure that the effect of better information through the covariance channel lowers the conditional equity premium. To generate the same effect when returns are countercyclical (EIS $\in (0.5, 1)$), we need a higher RA. In particular, we need the investors to have a sufficiently strong preference for early resolution of uncertainty that $(1-\kappa)+\gamma < 0$.

3. Two Special Cases

In general, we cannot determine the direction of the effect of better information on the conditional equity premium through the variance and covariance channels conjectured previously. Two important exceptions are:

Power Utility. With power utility, κ equals 1. So the conditional equity premium does not depend on the conditional variance of returns net of growth. Moreover, the coefficient on the conditional covariance term simplifies to γ, which is always positive. By Conjecture 2, the conditional equity premium is conjectured to decrease with the quality of the external signal if $\psi > 1$ (which implies $\gamma < 1$).

Moderate EIS and Relatively High RA. When $\frac{1}{2} < \psi < 1$ but the relative RA is sufficiently high ($\gamma > 1/(2\psi - 1)$), the conditional equity premium is conjectured to decrease in the quality of the external signal.

V. Numerical Results

The log-linearized approximation in the last section allows us to conjecture the direction of the effect of information quality on the equilibrium equity premium in some special cases. In this section we extend the analysis in several directions: First, we analyze the quantitative implications of information on stock returns for a particular parameter choice. Second, we show how the effect of information varies with the relative magnitude of the EIS and the RA parameters. The latter allows us to establish a parameter region where better information lowers the equity premium. Third, we provide a numerical analysis on the Sharpe (1966) ratio for a wide range of parameters.

A. Benchmark Calibration

In the first part of this section, we look at how the conditional equity premium changes with 2 variables: the current state uncertainty and the quality of the external signal. This lays the foundation for understanding the impact of information quality on the average equity premium.

Our benchmark parameterization follows Bansal and Yaron (2004): Investor RA is given by $\gamma = 10$, investor EIS is given by $\psi = 1.5$, and the time discount rate is given by $\beta = 0.9925$. The persistence of the estimated consumption process is lower than that of the process they postulate, so these parameter values do not allow us to match the empirical equity premium.[11]

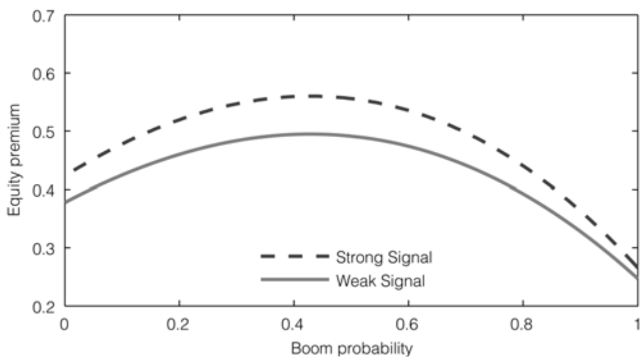
[11] To match both the empirical equity premium and the average risk-free rate for our benchmark EIS of 1.5, we would need to set γ and β to 375 and 0.9785, respectively (an interesting analysis on the magnitude of the Epstein and Zin (1989) utility parameters can be found in Campanale, Castro, and Clementi (2008)). A higher ψ increases the riskiness of equity by making returns more procyclical,

We start off by looking at the equity premium conditional on given beliefs about the state of the world for the benchmark calibration. The solid line in Figure 3 shows the conditional log return equity premium from equation (11) as a function of the probability that investors assign to the boom state when they have access only to the consumption signal ($h = 0$). The dashed line in the figure shows the conditional log return equity premium they demand when they also have access to a strong external signal ($h = 0.99$). Three facets of the figure are particularly noteworthy: First, the conditional log return equity premium is hump shaped in the probability investors assign to the boom state. Second, the conditional log return equity premium is lower in booms than in recessions. Third, for given current beliefs about the state of the economy, the conditional log return equity premium is uniformly higher when the investors also have access to the strong external signal. All 3 facets are related to the conditional variance of equity returns. In the benchmark calibration, the variance of equity returns enters equation (11) with a positive coefficient, so the conditional log return equity premium will be higher the higher the variance of equity returns. From equation (8) we know that the PD ratio is a function of the current beliefs about the state of the economy. The more beliefs are expected to be revised in the immediate future, the higher the conditional variance of equity returns.

FIGURE 3

Conditional Equity Premium and Signal Strength

Figure 3 gives the conditional log return equity premium ($E_t\left[r^e_{t+1}\right] - r^f_t + (1/2)\sigma^2_t(r^e_{t+1})$ in % per annum (p.a.)) predicted by the model as a function of the probability investors assign to the boom state. The dashed line is obtained with a very strong signal ($h = 0.99$); the solid line is for the case where investors only have access to the consumption signal ($h = 0$). The utility parameters are set to the benchmark values ($\gamma = 10$, $\psi = 1.5$, and $\beta = 0.9925$).



Even when investors only have access to the consumption signal, their beliefs typically gravitate quickly toward the true state of the economy. Periods when there is great uncertainty about the state of the economy are in expectation followed by learning about the true state of the economy, and such learning will lead to price movements. This contributes to the hump-shaped pattern for the equity

but even for very high values of ψ, extreme RA is needed for the model to match the postwar equity premium. If we use $\psi = 20$, which is an order of magnitude greater than what has been reported in the literature (see footnote 1), we still need an RA parameter of 295 and a time discount rate of 0.9815 to match the average risk-free rate and equity premium in the postwar data.

premium: When uncertainty is high, expected return volatility is also high and investors require a high premium to hold equity.

A related effect contributes to the fact that the equity premium is higher in recessions than in booms. The estimated average duration of a boom is about 4 years, whereas recessions on average last only about a year (see Table 3). In booms, investors do not expect to receive signals that will greatly change their beliefs about the state because the state itself is unlikely to change. In contrast, during recessions, the state itself is likely to switch, and investors know that such a switch will be reflected in the signals they receive. This makes the conditional volatility of equity returns higher in recessions than in booms.

As discussed previously, the quality of the external signal also affects the volatility of prices. When the signal is very strong, investors know that it will accurately reflect the state of the economy. If they are currently uncertain about the state of the economy, they expect to either believe firmly that they are in a boom after they receive the next period's signal or to believe firmly that they are in a recession. Both events entail revisions of their current beliefs and thus return volatility. Even when investors are very certain about the prevailing state of the economy, stronger signals increase the conditional volatility of returns. With a strong external signal, it is easier to detect switches in the underlying state of the economy, and such switches would entail larger price movements. The higher conditional return variance with a strong signal contributes to the upward shift of the conditional equity premium from the solid line to the dashed line in Figure 3.

The higher *conditional* log return equity premium (equation (11)) with high signal quality does not translate into a higher *average* log return equity premium, $\mathrm{E}[r_{t+1}^e - r_t^f + (1/2)\sigma_t^2(r_{t+1}^e)]$. This is because with a strong signal investors will typically be certain about whether they are in a boom or a recession state. From Figure 3, we know that this implies a relatively low conditional equity premium. With a weaker signal, investors will be more uncertain about the state of the economy. Since the conditional equity premium is hump shaped in the boom-state probability, this will drive up the average equity premium.

This is confirmed in Figure 4, which decomposes the average log return equity premium as a function of signal strength. Graphs A and B show the contribution of return variance and the covariance of returns and consumption growth rates to the equity premium, respectively, while Graph C shows the average log return equity premium. As we conjecture in Section IV.B.3, the variance of returns increases with the quality of the external signal, while the covariance of returns and consumption decreases with it. The net effect is an average log return equity premium, which increases with the quality of the external signal.

B. Other Parameterizations

1. EIS > 1

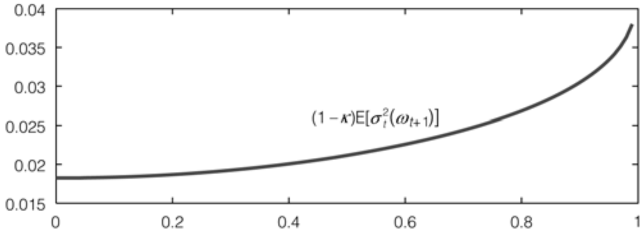
With $\psi > 1$, prices are procyclical and better information lowers the covariance of returns net of dividend growth and consumption growth rates, while, as always, a higher signal quality increases the variance of returns net of dividends. For this parameterization, the 2 forces pull in opposite directions, so we need to resort to numerical techniques to determine the total impact.

FIGURE 4

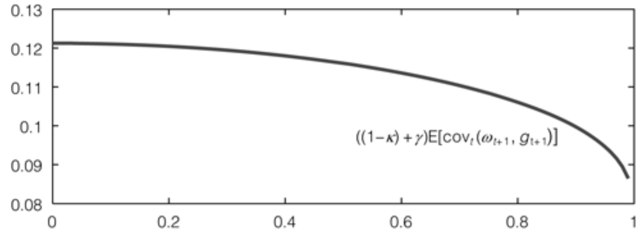
Effect of Information Quality on the Expected Equity Premium

Figure 4 decomposes the average log return equity premium ($\mathrm{E}[r^e_{t+1} - r^f_t + (1/2)\sigma^2_t(r^e_{t+1})]$ in % p.a.) as a function of signal strength. The signal strength is increasing along the *x*-axis, where 0 indicates a completely noisy signal and 1 a perfect signal. Graph A gives the contribution of the conditional variance of returns to the equity premium. Graph B gives the contribution from the conditional covariance of returns and consumption growth rates. The equity premium (Graph C) aggregates these 2 terms to the contribution from the conditional variance of consumption growth rates. The utility parameters are set to the benchmark values ($\gamma = 10$, $\psi = 1.5$, and $\beta = 0.9925$).

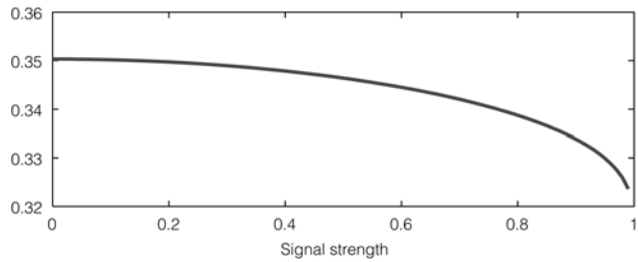
Graph A. Contribution of Variance



Graph B. Contribution of Covariance



Graph C. Equity Premium



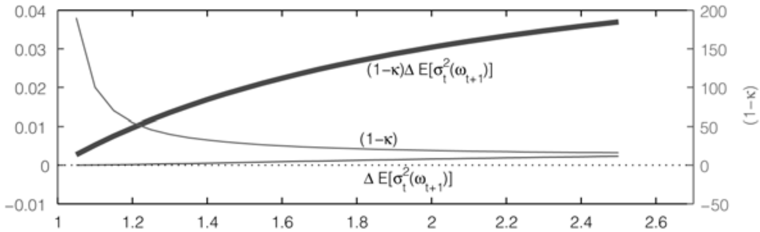
Graphs A and B of Figure 5 decompose the change in the average log return equity premium a representative investor would require when given access to a high-quality signal (h goes from 0.01 to 0.99). In these 2 graphs, the investor's RA parameter is set to $\gamma = 10$. As we move along the abscissa, the EIS parameter ψ of the representative investor varies from 1.05 to 2.5. The contribution of the reduction in the average conditional variance $\mathrm{E}[\sigma^2_t(\omega_{t+1})]$ is given in Graph A. As argued previously, information increases the variance of returns. For $\psi > 1$, this increases the required equity premium (Graph A). Better information quality also reduces the covariance between consumption growth and returns. This lowers the required equity premium (Graph B). The solid line in Graph C aggregates these terms with the contribution from the change in the consumption growth rate variance term, which is independent of ψ. The dashed lines in Graph C show the change in equity premium for alternative levels of RA. For all parameter values plotted, the net effect is an average log return equity premium that decreases with the quality of the external signal.

FIGURE 5

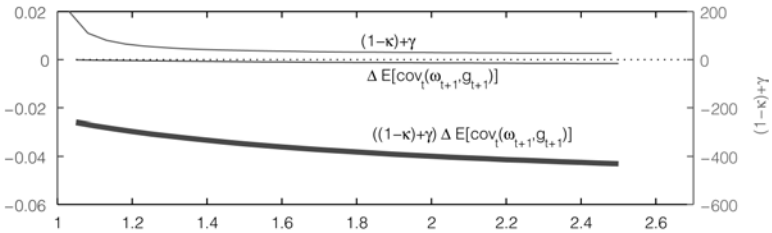
Improved Information and the Equity Premium (EIS $>$ 1)

Figure 5 decomposes the change in the average log return equity premium ($E[r^e_{t+1} - r^f_t + (1/2)\sigma^2_t(r^e_{t+1})]$ in % p.a.) that results from changing the strength of the external signal from $h = 0.01$ (very uninformative) to $h = 0.99$ (very informative). As we move along the abscissa, investors become increasingly tolerant to shifting consumption over time. In Graphs A and B, the other parameters are fixed to their benchmark values ($\gamma = 10$, $\beta = 0.9925$). Graph A shows the contribution to the equity premium that comes from the higher variance of returns, while Graph B shows the contribution from the change in the covariance between returns and consumption that results from a stronger signal. The solid line in Graph C aggregates these terms with the contribution from the change in the consumption growth rate variance term (which is independent of ψ). The dashed lines in Graph C show the change in equity premium for alternative levels of RA.

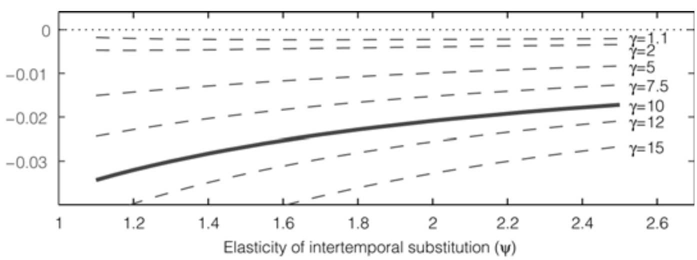
Graph A. Contribution of Change in Variance



Graph B. Contribution of Change in Covariance



Graph C. Change in Equity Premium



2. EIS $<$ 1

The result for $\psi > 1$ is important but not surprising, since $\psi > 1$ is the condition for procyclical prices. In the last section, we conjecture that the same effect obtains when $½ < \psi < 1$ and $\gamma > 1/(2\psi - 1)$. If these conditions are not satisfied, we cannot rely on the intuition-driven conjecture from the last section because the impact of a change in the information quality through the variance and covariance channels pulls the equilibrium equity premium in opposite directions.

Nevertheless, we can establish a parameter region where improved information quality lowers the equilibrium return to equity. One way to do so is to fix the level of RA and investigate how the effect of information on the equity premium changes as we vary the willingness of investors to substitute consumption over time.

Figure 6 illustrates this procedure and links the numerical solution to the discussion of the terms in the log-linear approximation. It decomposes the change in the average log return equity premium changes when agents are provided with a high-quality signal (h goes from 0.01 to 0.99). In Figure 6, γ is set to the benchmark value of 10, while we let ψ vary from 0.15 to 0.9.[12] The contribution of the reduction in variance on the required equity premium is given in Graph A. Better information increases the variance of returns, which, in most of the range plotted, lowers the required equity premium. The effect is reversed for EIS parameters below $1/\gamma$, or 0.1 in our benchmark parameterization.

Graph B of Figure 6 provides the analogous decomposition for the effect of the change in covariance between consumption and returns when the signal quality varies. As argued previously, returns covary less with consumption when the signal quality is high. For $\psi < 1$, the covariance is negative. So an increase in the signal quality leads to a less negative covariance. The lower ψ, the more pronounced the effect. The effect on the change in the covariance is shown by the bold line in Graph B. It is the product of the change in the covariance and its coefficient in equation (11). The gray area delimits the range where the effect of lower signal precision through this component leads to a lower required equity premium.

The net effect of the variance and covariance terms is given in Graph C of Figure 6. This is the sum of the 2 effects described previously and the effect of the variance of the dividend growth rates term, which is constant.

Repeating the same exercise as in Figure 6 for various levels of investor RA yields a locus that marks the boundary at which the effect of information quality on the average log return equity premium reverses. In Figure 7, we mark such points of intersection for a set of RA parameters with a solid line. It separates the 2 regions of interest: Above the line, the relation is negative; below it is positive. The dashed line sets out parameter combinations for the power utility case.

C. Information Quality and the Sharpe Ratio

In this section we extend our numerical analysis to the effect of information quality on the unconditional log return equity Sharpe (1966) ratio defined as the

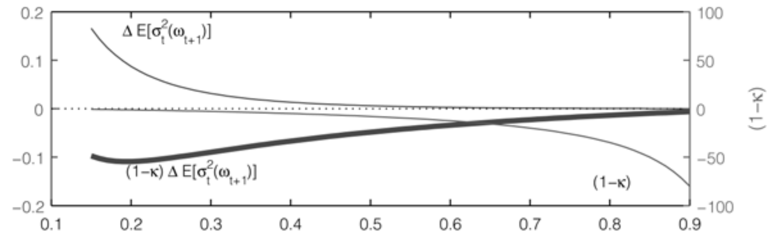
[12]We avoid going too close to 1: The returns-based Euler equation is not defined at $\psi = 1$, and there is no reason to expect the log-linearization to be a good approximation around this point.

FIGURE 6

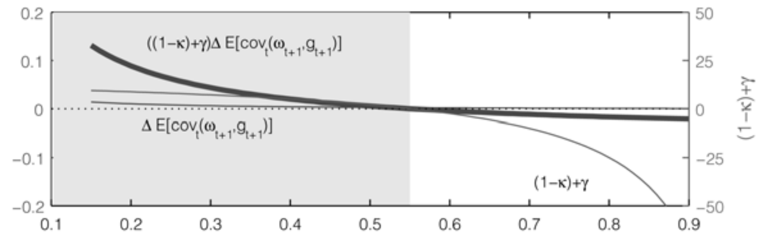
Improved Information and the Equity Premium (EIS < 1)

Figure 6 decomposes the change in the average log return equity premium ($\mathrm{E}[r_{t+1}^e - r_t^f + (1/2)\sigma_t^2(r_{t+1}^e)]$ in % p.a.) that results from changing the strength of the external signal from $h = 0.01$ (very uninformative) to $h = 0.99$ (very informative). As we move along the abscissa, investors become increasingly tolerant to shifting consumption over time, while the other utility parameters are fixed to their benchmark values ($\gamma = 10$, $\beta = 0.9925$). Graph A shows the contribution to the equity premium that comes from the higher return variance, while Graph B shows the contribution from the change in the covariance between returns and consumption that results from a stronger signal. The total effect (Graph C) aggregates these 2 terms with the contribution from the change in the variance of consumption growth rates.

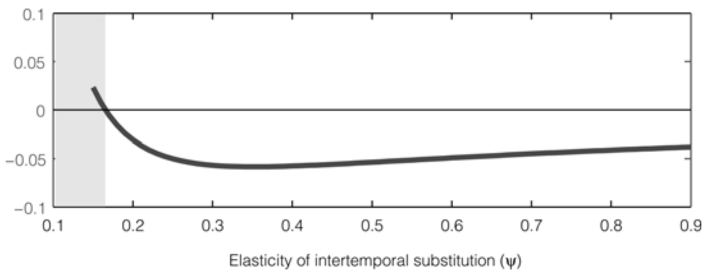
Graph A. Contribution of Change in Variance



Graph B. Contribution of Change in Covariance



Graph C. Change in Equity Premium



average log return equity premium divided by the unconditional variance of equity log returns

$$\text{(12)} \qquad \mathrm{SR} \quad = \quad \frac{\mathrm{E}\left[r_{t+1}^e - r_t^f + \frac{1}{2}\sigma_t^2\left(r_{t+1}^e\right)\right]}{\sigma\left(r_{t+1}^e\right)}.$$

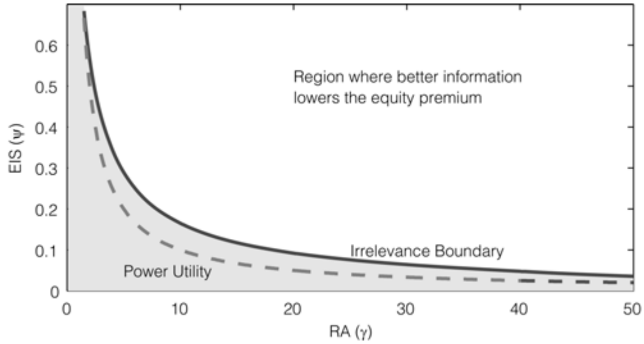
Obviously, the effect of information quality on the Sharpe ratio will depend on its effect on both the equity premium and the standard deviation of equity returns. It is helpful to decompose the unconditional variance of equity log returns as

$$\text{(13)} \qquad \sigma^2\left(r_{t+1}^e\right) \quad = \quad \sigma^2\left(g_{t+1}\right) + \sigma^2\left(\omega_{t+1}\right) + 2\mathrm{cov}\left(g_{t+1}, \omega_{t+1}\right).$$

FIGURE 7

Information Quality and Equity Premium by Parameter Regions

Figure 7 groups parameter constellations by the effect of better information on the average log return equity premium ($\mathrm{E}[r^e_{t+1} - r^f_t + (1/2)\sigma^2_t(r^e_{t+1})]$ in % p.a.). The shaded region depicts parameter constellations where better information increases the equity premium. For the domains considered, power utility specifications fall in this region. The time discount rate is set to 0.9925.



Consumption growth is exogenous, so information quality will only impact the denominator in equation (12) through the last 2 terms of equation (13). Graph A of Figure 8 plots the effect of increasing the precision of the external signal from $h = 0.01$ to $h = 0.99$ on the terms in equation (13) for different values of ψ when the other model parameters are kept at their benchmark values ($\gamma = 10$, $\beta = 0.9925$). We know from Proposition 1 that the PD ratio is constant at $\psi = 1$. At this point the variance of equity returns is not affected by information quality. As we move the EIS away from 1, the last 2 terms on the right-hand side of equation (13) come into play. The solid line gives the total effect of better information on the unconditional variance of equity log returns.

The solid line in Graph B of Figure 8 gives the corresponding change in the unconditional log return equity Sharpe (1966) ratio defined in equation (12). For moderate and high levels of ψ, better information lowers the Sharpe ratio, whereas for low levels of ψ, the better information quality actually leads to a higher Sharpe ratio for equity. The seemingly erratic behavior of this relation as $\psi \longrightarrow 0$ is due to the fact that the equity premium becomes negative for low levels of ψ.[13] In this case, an increase in the variance of returns increases the average Sharpe ratio by bringing it closer to 0. Increasing the coefficient of relative RA from the benchmark calibration magnifies the pattern. This is illustrated by the 2 other lines in Graph B. The response of the Sharpe ratio for $\gamma = 5$ is more subdued (dashed line), while the response for $\gamma = 15$ is more pronounced (dash-dotted line).

As we did for the equity premium in Figure 6, we repeat the exercise in Figure 8 for different levels of γ and collect the points where the impact of information quality on the unconditional log return equity Sharpe (1966) ratio

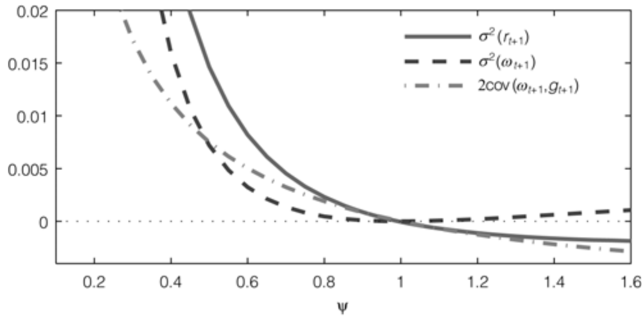
[13] At low levels of ψ the price-consumption ratio is sufficiently countercyclical to make equity a hedge against consumption risk. See the negative premiums reported in Table 4.

FIGURE 8

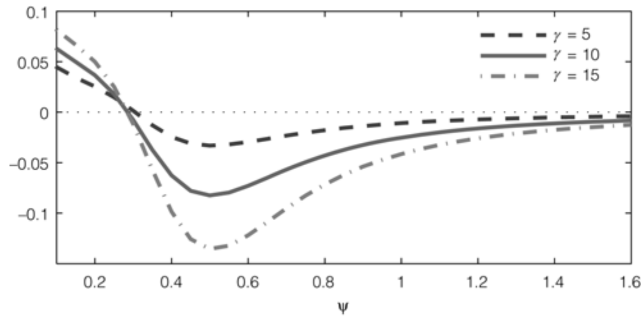
Improved Information and the Sharpe Ratio

Figure 8 decomposes the change in the unconditional log return Sharpe (1966) ratio (as defined in equation (12)) that results from changing the strength of the external signal from $h = 0.01$ (very uninformative) to $h = 0.99$ (very informative). As we move along the abscissa, investors become increasingly tolerant to shifting consumption over time. In Graph A, the other parameters are fixed to their benchmark values ($\gamma = 10$, $\beta = 0.9925$). Graph A shows the effect of better information on the variance of returns (solid line). The effect is decomposed into the variance of returns net of dividend growth and their covariance with consumption growth. Graph B shows the corresponding change in the Sharpe ratio for alternative levels of RA.

Graph A. Change in Second Moments



Graph B. Change in Sharpe Ratio



defined in equation (12) changes sign. These points are plotted as the solid line in Figure 9. It separates the 2 regions of interest: For parameter combinations that are above the line, better information leads to a lower Sharpe ratio, while for the parameter combinations in the shaded region below the line, better information leads to a higher Sharpe ratio. The dashed line sets out parameter combinations for the power utility case. The link between the EIS and RA parameters places us in the shaded region when the RA is high. Here, the counterintuitive effect of better information quality on the log return equity premium under power utility is inherited by the Sharpe ratio.

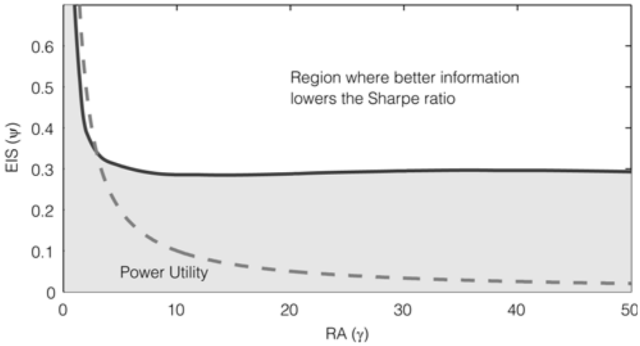
D. Unbounded Equity Premium and Sharpe Ratio

As noted in the introduction, another feature that Veronesi (2000) identified for the power utility case in this setting is that the equity premium is bounded above at a low value when signals are noisy. This seems to aggravate the equity

FIGURE 9

Information Quality and Sharpe Ratio by Parameter Region

Figure 9 groups parameter constellations by the effect of better information on the unconditional log return Sharpe (1966) ratio as defined in equation (12). The shaded region gives parameter constellations where better information leads to a higher Sharpe ratio. The time discount rate is set to 0.9925. The dashed line gives parameter combinations where the utility function simplifies to a power utility function.



premium puzzle, because the model predicts a negative equity premium for high levels of RA.

The solid line in Graph A of Figure 10 replicates that in Panel B of Figure 2 from Veronesi (2000). The external signal is set to be moderately informative ($h = 0.5$). The line displays a well-defined global maximum. Such a maximum makes it even more difficult to solve the equity premium puzzle of Mehra and Prescott (1985), because increasing the RA parameter would lower the equity premium beyond this point. As a means of comparison, the graph also plots the average log return equity premium with the Epstein-Zin (1989) preferences. In this setup, increasing the RA parameter does not affect the EIS and hence leaves the cyclicality of returns largely unaffected. The upshot is that for most values of ψ a practically linear and increasing relation between the average log return equity premium and RA obtains. Only for very small values of ψ is this relation reversed. For such small values, returns become negatively correlated with the pricing kernel. Thus, the more risk-averse investors are, the greater the returns they are willing to give up to hold such a claim.[14]

Graph B of Figure 10 shows that the relation between the RA parameter and the average log return equity premium is reflected in the unconditional log return Sharpe (1966) ratio. With power utility (solid line), the Sharpe ratio is hump shaped in the RA parameter, while with the Epstein-Zin (1989) preferences (dashed lines) the Sharpe ratio changes in a seemingly linear way with the RA parameter.

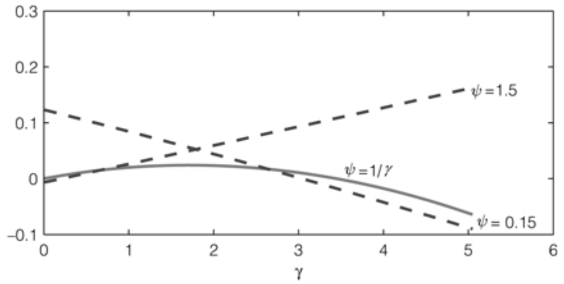
[14]In general, the average log return equity premium is not 0, even when investors have an RA parameter of 0. The exception is the curve graphing the power utility case where the average log return equity premium at $\gamma = 0$ is exactly 0. Mathematically, we can see that this holds by noting that equation (11) equals $-\big(1/(\psi - 1)\big)(\sigma_\omega^2 + \sigma_{\omega,g})$ at $\gamma = 0$. Unless we are in the power utility case, where $\psi \longrightarrow +\infty$ as $\gamma \longrightarrow 0$, the average log return equity premium will not be 0 at $\gamma = 0$.

FIGURE 10

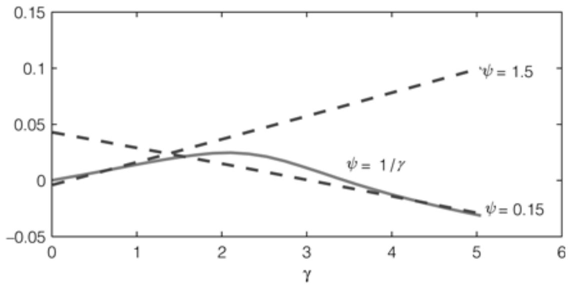
Equity Premium and Utility Parameters

Figure 10 plots the average log return equity premium (Graph A) and the unconditional log return Sharpe (1966) ratio (Graph B) against the coefficient of relative RA with the Epstein-Zin (1989) preferences (dashed lines) and power utility (solid line). The expected log return equity premium is calculated as $E[r^e_{t+1} - r^f_t + (1/2)\sigma^2_t(r^e_{t+1})]$, and the Sharpe ratio is calculated using equation (12). With the Epstein-Zin (1989) preferences, the EIS parameter can be held fixed. With power utility, the EIS parameter is given by the inverse of RA. All lines are plotted for a time discount factor β of 0.9925 and assuming an exogenous signal that is moderately informative ($h = 0.5$).

Graph A. Equity Premium



Graph B. Sharpe Ratio



The patterns from Figure 10 are confirmed for a larger set of parameter values in Table 4. Except for the lowest level of the EIS parameter (leftmost column), there is a positive relation between γ and both the average log return equity premium and the unconditional log return Sharpe (1966) ratio. This relation is stronger the further ψ is away from 1, reflecting that the EIS parameter governs the cyclicality of returns.

VI. Conclusions

This paper focuses on the implications of changes in the quality of information on asset prices in a pure exchange economy. Matching empirical figures with model predictions has been a challenging aim ever since the seminal contribution of Mehra and Prescott (1985). When variations in information quality are introduced, the model predictions become even more puzzling. Veronesi (2000) has shown that if investors maximize a power utility function, the required risk premium is increasing in the quality of information. He also shows that, in this case, there is a strict and small upper bound for the attainable equity premium.

TABLE 4

Predicted Equity Premium and Sharpe Ratio

Table 4 reports the average log return equity premium $(E[r^e_{t+1} - r^f_t + (1/2)\sigma^2_t(r^e_{t+1})])$ and the Sharpe (1966) ratio for a range of RA (γ) and EIS (ψ) parameters. The Sharpe ratio is computed according to equation (12). All values are in % p.a., computed with a time discount factor of 0.9925 and assuming a signal that is moderately informative ($h = 0.5$).

	ψ					
γ	0.25	0.50	0.75	1.20	1.75	2.50
Panel A. Equity Premium						
2	0.01	0.02	0.04	0.05	0.06	0.07
4	–0.02	0.05	0.09	0.12	0.14	0.15
10	–0.13	0.13	0.23	0.32	0.36	0.39
25	–0.59	0.29	0.62	0.87	1.01	1.10
50	–1.64	0.53	1.29	1.87	2.18	2.38
75	–2.24	0.90	1.96	2.76	3.19	3.46
100	–2.28	1.38	2.60	3.52	4.00	4.31
Panel B. Sharpe Ratio						
2	0.00	0.02	0.03	0.04	0.04	0.04
4	–0.01	0.05	0.07	0.08	0.08	0.08
10	–0.06	0.12	0.18	0.21	0.21	0.21
25	–0.26	0.26	0.49	0.56	0.58	0.58
50	–0.69	0.48	1.03	1.20	1.23	1.25
75	–1.02	0.82	1.56	1.78	1.83	1.85
100	–1.17	1.27	2.04	2.28	2.34	2.36

We generalize Veronesi's (2000) model by using the Epstein-Zin (1989) utility specification. This allows us to revisit the relation between information quality and the equity premium for a broader set of parameter combinations. We provide both an analytical and a numerical analysis of the relation and find a large region of parameter values for which his result is overturned. We also find that the upper bound on the equity premium is an artifact of the restriction embedded in the power utility specification and that there is no apparent local maximum on the equity premium with the Epstein-Zin (1989) preferences.

The parameter region where the equity premium is decreasing in signal quality contains both cases where investors have an EIS greater than 1 and an EIS smaller than 1, provided that they are sufficiently risk averse. When the EIS is less than 1, the interplay between the utility parameters switches the signs of the relevant second moments, allowing the model to predict an equity premium that is decreasing in signal quality. When the EIS is greater than 1, the result is mainly driven by the capability of the model to generate procyclical prices. The degree to which this procyclicality translates into a positive covariance between consumption and returns, and hence high risk premiums, depends on the quality of the signals available to investors. The better the external information available, the less prices will be driven by the information embedded in consumption growth rates, and the smaller the covariance will be.

Appendix A. Computational Details

1. Discretized Dynamics

For a given model economy, equation (8) relates the current price-consumption ratio to expectations of future consumption growth rates and future price-consumption ratios. The only state variable the time t consumption-price ratio depends on is the investors' posterior boom probability given their t information set. Lacking a functional form for the price-consumption ratio, we solve for price-consumption ratios in a discretized version of the model.

Denote the posterior boom probability of the representative investor, given the time t information set, by ξ_t and his posterior boom probability, given the time t information set less the external signal, by ξ_t^c. Conceptually, we think of the investor as first observing the consumption growth rate and updating his belief based on this realization and then observing the external signal: After observing the consumption growth rate, his posterior boom probability is ξ_{t+1}^c; after observing the external signal, it is updated to ξ_{t+1}. We discretize the interval $[0, 1]$, the support of the posterior boom probability into n equally spaced grid points; ξ_i, the ith grid point, is at $(i-1)/(n-1)$.

Let $F(\xi_{t+1}^c|\xi_t)$ denote the 1-period-ahead distribution function for the posterior boom probability given the consumption signal. We start by approximating this distribution function with the discrete distribution

$$\hat{F}(\xi_i^c|\xi_t=\xi_j) \quad = \quad \begin{cases} \big(F(\xi_i^c|\xi_t=\xi_j)+F(\xi_{i+1}^c|\xi_t=\xi_j)\big)/2, & 1\leq i\leq n \\ 1, & i=n. \end{cases}$$

Analogously, let $G(\xi_{t+1}|\xi_{t+1}^c)$ denote the distribution of the posterior boom probabilities after the consumption signal is observed but before the external signal is observed. We approximate this distribution with the distribution function

$$\hat{G}(\xi_i|\xi_{t+1}^c=\xi_j) \quad = \quad \begin{cases} \big(G(\xi_i|\xi_{t+1}^c=\xi_j)+G(\xi_{i+1}|\xi_{t+1}^c=\xi_j)\big)/2, & 1\leq i\leq n \\ 1, & i=n. \end{cases}$$

Let $\hat{\Theta}^c$ be the matrix of transition probabilities between the grid points given only the consumption signal. The element $\hat{\Theta}_{i,j}^c$ gives the probability of the representative investor's posterior boom probability at time $t+1$ being ξ_i, given that his posterior probability at time t was ξ_j and the model parameters. We compute the elements of $\hat{\Theta}^c$ by

$$\hat{\Theta}_{i,j}^c \quad = \quad \begin{cases} \hat{F}(\xi_i|\xi_t=\xi_j), & i=1 \\ \hat{F}(\xi_i|\xi_t=\xi_j)-\hat{F}(\xi_{i-1}|\xi_t=\xi_j), & 2\leq i\leq n. \end{cases}$$

Analogously, let $\hat{\Theta}^y$ be the matrix of transition probabilities between posterior boom probabilities given only the consumption signal and the posterior boom probabilities given both the external signal and the consumption signal. The elements of Θ^y are computed in the same manner as those of Θ^c, using the approximate conditional distribution $\hat{G}$ instead of $\hat{F}$.

Finally, we collect the transition probabilities $\Pr(\xi_{t+1}=\xi_i \mid \xi_t=\xi_j)$ in the matrix $\hat{\Theta}$ computed as

$$\hat{\Theta} \quad = \quad \hat{\Theta}^y\hat{\Theta}^c.$$

The ergodic discretized distribution of ξ solves

$$\pi^\star \quad = \quad \hat{\Theta}\pi^\star \quad \text{and} \quad 1 \quad = \quad \sum_s \pi^\star(s).$$

We solve for the gross consumption growth rate that takes investor beliefs from $\xi_t=\xi_j$ to ξ_{t+1}^c and collect them in the matrix $\hat{C}$:

$$\hat{C}_{i,j} \quad = \quad \mathcal{F}(\xi_{t+1}^c=\xi_i,\xi_t=\xi_j).$$

In our discretized economy, the vector of state price-consumption ratios $\hat{W}$ now solves:

$$\hat{W}_i \quad = \quad \beta^\kappa\sum_{j=1}^n\sum_{k=1}^n\Theta_{j,k}^y\Theta_{k,i}^c(1+\hat{W}_j)^\kappa\hat{C}_{k,i}^{1-\gamma}.$$

Equity returns depend on both the consumption signal and on the external signal. If the consumption signal takes the investor's beliefs from ξ_j to state ξ_k and the external signal takes the investor's beliefs to ξ_i, we take the realized return to equity to be

$$R^e(\xi_{t+1} = \xi_i, \xi^c_{t+1} = \xi_k, \xi_t = \xi_j) \quad = \quad \frac{1 + \hat{W}_i}{\hat{W}_j}\hat{C}_{j,k}.$$

2. Financial Statistics

The conditional gross risk-free rate and the gross expected return to equity in the model can be calculated using

$$\begin{aligned} R^f_t(\xi_t = \xi_j) &= \left(\beta^\kappa \sum_{i=1}^{n}\sum_{k=1}^{n} \hat{\Theta}^y_{i,k}\hat{\Theta}^c_{k,j}\hat{C}_{k,j}\left(\frac{1+\hat{W}_i}{\hat{W}_j}\right)^{\kappa-1}\right)^{-1}, \\ \mathrm{E}_t\left[R^e_{t+1} \mid \xi_t = \xi_j\right] &= \sum_{i=1}^{n}\sum_{k=1}^{n} \hat{\Theta}^y_{i,k}\hat{\Theta}^c_{k,j}\frac{1+\hat{W}_i}{\hat{W}_j}\hat{C}_{k,j}. \end{aligned}$$

The conditional moments of the variables in the log-linear approximation are found by weighing their values in each of the possible paths with the probability of the paths:

$$\begin{aligned} \mathrm{E}\left[\omega_{t+1} \mid \xi_t = \xi_j\right] &= \sum_{i=1}^{n} \hat{\Theta}_{i,j}\log(1+\hat{W}_i) - \log \hat{W}_j, \\ \mathrm{E}\left[\omega^2_{t+1} \mid \xi_t = \xi_j\right] &= \sum_{i=1}^{n} \hat{\Theta}_{i,j}(\log(1+\hat{W}_i) - \log \hat{W}_j)^2, \\ \mathrm{E}\left[g_{t+1} \mid \xi_t = \xi_j\right] &= \sum_{i=1}^{n} \hat{\Theta}^c_{i,j}\log \hat{C}_{i,j}, \\ \mathrm{E}\left[g^2_{t+1} \mid \xi_t = \xi_j\right] &= \sum_{i=1}^{n} \hat{\Theta}^c_{i,j}(\log \hat{C}_{i,j})^2, \\ \mathrm{E}\left[g_{t+1}, \omega_{t+1} \mid \xi_t = \xi_j\right] &= \sum_{i=1}^{n}\sum_{k=1}^{n} \hat{\Theta}^y_{i,k}\hat{\Theta}^c_{k,j}(\log(1+\hat{W}_i) - \log \hat{W}_j + \log \hat{C}_{k,j}). \end{aligned}$$

The conditional variances and covariances in equation (11) are then found by the general formulas

$$\begin{aligned} \sigma^2_t(x_{t+1}) &= \mathrm{E}_t[x^2_{t+1}] - \mathrm{E}_t[x_{t+1}]^2, \\ \mathrm{cov}_t\,(x_{t+1}, y_{t+1}) &= \mathrm{E}_t[x_{t+1}, y_{t+1}] - \mathrm{E}_t[x_{t+1}]\mathrm{E}_t[y_{t+1}]. \end{aligned}$$

The reported average log equity premium is found by weighing the conditional equity premium for each of the belief states ξ_i with its ergodic probability or

$$\mathrm{E}\left[r^e_{t+1} - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right] \quad = \quad \sum_{i=1}^{n} \pi^\star_i \left(\mathrm{E}\left[r^e_{t+1} \mid \xi_t = \xi_i\right] - r^f_t + \frac{1}{2}\sigma^2_t(r^e_{t+1})\right).$$

The reported Sharpe (1966) ratio is found by dividing the average equity premium by the unconditional variance of log equity returns:

$$\sigma^2(r^e_{t+1}) \quad = \quad \sigma^2(\omega_{t+1}) + \sigma^2(g_{t+1}) + 2\mathrm{cov}\,(g_{t+1}, \omega_{t+1}).$$

Appendix B. Proofs and Derivations

Proof of Proposition 1. Denoting the wealth (assets) of the representative investor by A_t, we rewrite the value function of the representative investor as

$$V_t(A_t) = \left((1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \mathcal{R}(V_{t+1}(A_{t+1}))^{1-\frac{1}{\psi}} \right)^{\frac{1}{1-\frac{1}{\psi}}},$$

where $A_{t+1} = (A_t - C_t) R_{t+1}^e$. Taking the first-order condition for optimal consumption C_t, rearranging, and using the fact that the risk adjustment $\mathcal{R}$ is homogeneous of degree 1 gives

$$\frac{A_t}{C_t} = \frac{1}{1-\beta} \left(\frac{V_t}{C_t} \right)^{1-\frac{1}{\psi}}.$$

This equation links the wealth-consumption ratio to the scaled continuation value of the representative investor. In equilibrium, the representative investor's portfolio consists of 1 unit of the claim to the aggregate consumption, so the left-hand side of the equation above equals the quantity W_t in the text. We now need to establish that the continuation value V_t is monotonically increasing in the boom probability. Let ξ and ξ' be 2 possible boom probabilities with $\xi < \xi'$. Given the process assumption and $\theta_1, \theta_2 > 0.5$, the distribution of future consumption conditional on ξ is stochastically dominated by the distribution of future consumption conditional on ξ'. It follows that the continuation value V_t is higher under ξ' than under ξ. This establishes that V_t is increasing in the boom probability if the 2 states are sufficiently persistent. It follows immediately that W_t is increasing in the boom probability if $\psi > 1$ and that it is decreasing in the boom probability if $\psi < 1$. For $\psi = 1$, the exponent on the right-hand side of the last equation is 0, making W_t constant and equal to $1/(1-\beta)$. □

Proof of Proposition 2. Here and henceforth we define $g_{t+1} = \log(C_{t+1}/C_t)$. In our setting, the return to any asset will obey the Euler equation $\mathrm{E}_t\left[M_{t+1} R_{t+1}\right] = 1$. When g_{t+1} and r_{t+1}^e are conditionally jointly log normal, we can log-linearize the Euler equations for the return to equity (R_e) and the risk-free rate (R_b). We obtain the following system of equations:

$$\text{(B-1)} \quad \begin{aligned} 0 &= \kappa \log(\beta) - \frac{\kappa}{\psi} \mathrm{E}_t\left[g_{t+1}\right] + \kappa \mathrm{E}_t\left[r_{t+1}^e\right] \\ &\quad + \frac{1}{2}\left[\left(\frac{\kappa}{\psi}\right)^2 \sigma_t^2(g_{t+1}) + \kappa^2 \sigma_t^2(r_{t+1}^e) - \frac{2\kappa^2}{\psi} \mathrm{cov}_t\left(g_{t+1}, r_{t+1}^e\right)\right], \\ 0 &= \kappa \log(\beta) - \frac{\kappa}{\psi} \mathrm{E}_t\left[g_{t+1}\right] + (\kappa - 1)\, \mathrm{E}_t\left[r_{t+1}^e\right] + r_t^f \\ &\quad + \frac{1}{2}\left[\left(\frac{\kappa}{\psi}\right)^2 \sigma_t^2(g_{t+1}) + (\kappa-1)^2 \sigma_t^2(r_{t+1}^e) - \frac{2\kappa(\kappa-1)}{\psi} \mathrm{cov}_t\left(g_{t+1}, r_{t+1}^e\right)\right], \end{aligned}$$

where $\sigma_t^2(g_{t+1})$ and $\sigma_t^2(r_{t+1}^e)$ denote the conditional variance of the consumption growth rate and the return to equity, respectively and $\mathrm{cov}_t\left(g_{t+1}, r_{t+1}^e\right)$ denotes their conditional covariance. Lowercase letters denote logged variables. Taking the difference between the 2 equations, we get[15]

$$\text{(B-2)} \qquad \mathrm{E}_t\left[r_{t+1}^e\right] - r_t^f + \frac{1}{2}\sigma_t^2(r_{t+1}^e) = (1-\kappa)\, \sigma_t^2(r_{t+1}^e) + \frac{\kappa}{\psi} \mathrm{cov}_t\left(g_{t+1}, r_{t+1}^e\right),$$

that is, the equity premium plus a Jensen's (1906) inequality term.

[15]This corresponds to equation (8.3.7) of Campbell, Lo, and MacKinlay ((1997), p. 320).

Now we can use the definition of log returns, $r_{t+1}^{e} = \omega_{t+1} + g_{t+1}$, to calculate the second moments in equation (B-2):

$$\text{(B-3)} \qquad \begin{aligned} \sigma_t^2(r_{t+1}^e) &= \sigma_t^2(\omega_{t+1}) + \sigma_t^2(g_{t+1}) + 2\text{cov}_t(\omega_{t+1}, g_{t+1}), \\ \text{cov}_t(g_{t+1}, r_{t+1}^e) &= \sigma_t^2(g_{t+1}) + \text{cov}_t(\omega_{t+1}, g_{t+1}). \end{aligned}$$

The expression for the equity premium follows by substituting equation (B-3) in equation (B-2). □

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