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RESEARCH ARTICLE

Asset pricing and the role of macroeconomic volatility

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Abstract Standard Real Business Cycle (RBC) models are well known to generate counter-factual asset pricing implications. This study provides a simple extension to the prior literature by studying an economy that follows a regime-switching process in conjunction with Epstein–Zin preferences for consumers. We provide a detailed theoretical and numerical analysis of the model's predictions. We also show that a reasonable parameterization of our model conveys financial figures in line with US postwar data. Furthermore, we provide evidence in support of modeling a regime-dependent macroeconomic risk.

Keywords Asset pricing · Real Business Cycle Models · Recursive preferences · Markov switching models

JEL Classification G12 · E32 · E23

1 Introduction

We study an economy that switches between booms and busts where technological shocks follow a hidden two-state Markov chain in conjunction with recursive preferences for consumers. Our work is closely related with the main literature on asset pricing with a non-trivial production side.[1] The study contributes a novel theoretical

[1] For an exhaustive analysis on the role of asset prices in RBC models see Lettau (2003).

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framework where a sizable equity premium can be obtained without imposing any kind of rigidity on the production side of the model (e.g. Boldrin et al. 2001; Jermann 1998) and without requiring an implausibly high value for the risk aversion of agents (e.g. Danthine et al. 1992; Rouwenhorst 1995; Tallarini 2000).

We show that, when the role of macroeconomic risk is taken into account, the model can replicate the US postwar financial figures. This provides support in favor of modeling the macroeconomic volatility as regime-dependent in order to examine the two decades of "great moderation" which started in the mid-80s, and the recent increase in the macroeconomic risk.

By building on Lettau et al. (2008), where the role of macroeconomic risk in an endowment economy is studied, we adopt a recursive utility specification for the consumption side, referred to as Epstein–Zin preferences (Epstein and Zin 1989, 1991; Weil 1989). Two reasons drive this choice. First, this form of utility function is widely used in the latest asset-pricing research (see Bansal and Yaron 2004; Campbell and Viceira 2001; Campbell et al. 2003; Brandt et al. 2004; Guvenen 2006, among others). Second, since Epstein–Zin preferences nest the power utility function as a special case, these are particularly useful to provide a closer comparison with the standard models based on the power utility specification.

In RBC literature, the first analysis on asset prices (Danthine et al. 1992; Rouwenhorst 1995), while unsuccessful in explaining the behavior of returns over the business cycle, provided useful insights on what would be a necessary ingredient of a successful model. Along this line, to improve the capability in explaining financial figures, the main literature on asset pricing with a non-trivial production side (Jermann 1998; Boldrin et al. 2001), introduced some form of rigidity in the model. While both Boldrin et al. (2001) and Jermann (1998) specify a habit utility for consumers, the former relies on a limited mobility of factors of production and the latter introduces capital adjustment costs.[2]

The assumption of a recursive but time non-separable felicity function is not novel in this literature. Tallarini (2000) and more recently, Gomes and Michaelides (2008) assume an Epstein–Zin utility function, and both document the inability to generate reasonable implied returns without introducing some production frictions in the model. Kaltenbrunner and Lochstoer (2010); Croce (2012), and Malkhozov (2012) study the role of long-run risk in a production setting, while Ai (2010) is additionally concerned with the equity premium implications of this class of models.

This study takes a different approach on the production side of the model, and instead of imposing any kind of restriction, it provides a simple extension of a standard RBC model where the economy switches between booms and busts. This is accomplished by letting the economy follow a hidden Markov chain. Most of the literature focuses on the implication of a Markov switching process in the conditional mean and in the volatility of the endowment process.[3]

[2] A different approach can be found in Cochrane (1991) who evaluates asset pricing implications from the producer's first order conditions.

[3] Regime switching is widely used in economics since the seminal contribution by Hamilton (1989). In particular, in the asset pricing literature, the implications of a Markov switching process in the conditional

This study differs from the ones mentioned above in that here regimes are introduced via the production side by allowing technology shocks to follow a latent two-state process both in the mean and the volatility.[4]

The role played by the volatility of underlying state of the economy in determining returns is the key to understand our results. Intuitively, if we shut down the regime on the volatility, we prevent the agent from entering a "high risk" regime (i.e. the high volatility regime), but we also deprive her of the possibility of entering a "low risk" regime. This creates an economy with a smoother path for the consumption claim. Such a path is appealing to the agents, and this would push down the price of the risk-free asset which would increase its return and reduce the equity premium. Therefore, an economy with a volatility regime gives a higher incentive to agents to use the risk-free asset to transfer consumption overtime, pushing its prices up, lowering the risk-free return and thus increasing the equity premium.

The remainder of the paper is organized as follows: Sect. 2 introduces the general model, derives equilibrium asset prices, and analyzes the determinants of the equity premium predicted by the model. Section 3 discusses model calibration and provides numerical analysis of asset returns' properties over the business cycle. Section 4 concludes the paper. The proofs, algebraic derivations, and additional results are provided in "Appendix".

2 Model

A standard production economy with two actors is considered. Consumers are modeled via a representative, risk-averse agent who derives utility from consumption, while the production side is modeled through a standard representative firm that maximizes its shareholders' value. There are two securities in the economy: a riskless bond that agents can use for transferring their wealth to the future and equity, which provides a claim on the firm's profits.

2.1 Consumers

The representative agent has preferences defined over current consumption and future utility. Following Epstein and Zin (1989, 1991), the utility function is defined recursively by:

Footnote 3 Continued
mean of the endowment process are analyzed by Cecchetti et al. (1990); Kandel and Stambaugh (1991), Cecchetti et al. (1993); Abel (1994, 1999). Recently, the time series properties of the second moments gained popularity in this framework: by setting up an equilibrium economy where the endowment process follows a latent two state regime switching process, Veronesi (1999) shows a better explanatory power of volatility clustering than a model without regimes. In the same setting, Whitelaw (2000) introduces time-varying transition probabilities between regimes, finding a complex nonlinear relation between expected returns and stock market volatility. A recent contribution that studies the impact of regime switches in the volatility of the endowment process is in Lettau et al. (2008).

[4] In a different setting Cagetti et al. (2002) model the technology shocks as a Markov switching model in the first moment.

$$U(C_t, \mathbb{E}_t(U_{t+1})) = \left[(1-\beta)C_t^{\frac{1-\gamma}{\alpha}} + \beta(\mathbb{E}_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{1-\gamma}}, \tag{1}$$

where C_t indicates aggregate consumption, β is the time preference parameter, and $\alpha \equiv (1-\gamma)/(1-1/\psi)$, with $\gamma > 0$.

Parameter γ is the coefficient of relative risk aversion (RA), while the elasticity of intertemporal substitution (EIS) is given by ψ.[5]

2.2 Firms and technology

For each period, the firm has to decide on the amounts of input factors it needs to employ.

In particular, there is only one traded good which is produced through a constant return to scale technology. Analytically, production can be described using a Cobb–Douglas function with human and physical capital as factors

$$Y_t = A_t K_t^{\theta} H_t^{1-\theta}, \tag{2}$$

where θ is the share of physical capital.

The human capital H evolves according to

$$H_{t+1} = (1-\delta_H)H_t + E_t, \tag{3}$$

where δ_H is its depreciation rate, and E_t is the investment in education.[6]

The capital stock's K evolution is governed by

$$K_{t+1} = (1-\delta_K)K_t + I_t, \tag{4}$$

where δ_K is its depreciation rate, and I_t indicates capital investment.

The resource constraint for this economy is written as

$$C_t + I_t + E_t \leq Y_t. \tag{5}$$

If we consider the productivity shock (A) in a regime-switching model, we can express its law of motion as a process with stochastic parameters that depend on the state of the economy

$$\Delta \log A_t = \mu(s_t) + \sigma(s_t)\varepsilon_t, \tag{6}$$

[5] An interesting feature of this utility function specification is that it nests the power utility. In fact, when $\gamma = \frac{1}{\psi}$ Eq. 1 can be solved forward to get the standard power utility function.

[6] As in Barro and Sala-I-Martin (2004) we can think of human capital as the number of workers multiplied by the human capital of a typical worker. We decide to explicitly model the human capital to assess the role of education expenses during the business cycles.

where μ and σ define the mean and the volatility of the process, and s_t indicates the state of the economy. We assume that s_t follows a hidden Markov chain with transition probabilities matrix P (see Hamilton 1989). The evolution of the state of the economy in terms of state beliefs (ξ_{t+1}) can be expressed as realizations of the equation

$$\xi_{t+1} = P\xi_t + \epsilon_t. \tag{7}$$

The agents cannot directly observe the state of the economy, s_t, and they have to rely on interpreting external signals. The agents update their belief according to the posterior probabilities computed as

$$\hat{\xi}_{t+1|t} = P \frac{\hat{\xi}_{t|t-1} \odot \zeta_t}{\mathbf{1}'\left(\hat{\xi}_{t|t-1} \odot \zeta_t\right)}, \tag{8}$$

where $\odot$ denotes the Hadamard product, ζ_t is a vector that stacks the conditional densities of the technological shocks' growth rates

$$\zeta_t = \begin{bmatrix} f\left(\Delta \log A_t \mid s_t = 1, \Omega_{t-1}\right) \\ \vdots \\ f\left(\Delta \log A_t \mid s_t = n, \Omega_{t-1}\right) \end{bmatrix} \tag{9}$$

Here, the density of $\Delta \log A_t$ conditional on state s_t is defined as:

$$f\left(\Delta \log A_t \mid s_t = i, , \Omega_{t-1}\right) = \frac{1}{\sqrt{2\pi}\sigma(s_t)} \exp\left\{-\frac{(\Delta \log A_t - \mu(s_t))^2}{2\sigma(s_t)^2}\right\}, \tag{10}$$

where Ω denotes the information set which contains the population parameters as well as the unconditional probability of the states s.

2.3 The firm's problem

The management of the representative firm maximizes the firm value through optimal investment given the current capital stock, the level of human capital hired, the current level of technology, and the stochastic discount factor.

In this economy, the firm's optimality condition for investment is summarized by the existence of a pricing kernel (Q) for pricing the investment return. In other words, the equality $\mathbb{E}_t\left[Q_{t+1}R^I_{t+1}\right] = 1$ holds true for the investment return (R^I) defined as $\theta\frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_K)$ (see "Appendix 1").

2.3.1 Asset prices

Next, we derive the equilibrium asset prices implied by the model. We examine two different types of assets: a one-period asset that yields one unit of consumption (i.e. a "risk-free" asset) and a claim to the physical capital (i.e. an "equity asset").

Epstein and Zin (1989) show that the stochastic discount factor for the case of utility over consumption is given by

$$Q_{t+1} = \beta^{\alpha} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\alpha}{\psi}} (R_{c,t+1})^{\alpha-1}, \tag{11}$$

where $R_{c,t+1}$ is the equilibrium gross return to consumption claim between t and $t+1$.

Then, with the assumed production technology (see Restoy and Rockinger 1994), if firms only use retained earnings to finance their capital the equity holder gets a dividend $D_t = \frac{\partial Y_t}{\partial K_t} K_t - I_t = \theta Y_t - I_t$. This implies that the unlevered stock market of this economy represents the value of the capital stock. Given the above dividends' expression, it is easy to show the following:

Remark 1 When the equity asset is considered,

a) the gross dividends' growth rate can be expressed as

$$\frac{D_{t+1}}{D_t} = \frac{A_{t+1}}{A_t} \left[\frac{K_{t+1}}{K_t} \right]^{\theta} \left[\frac{H_{t+1}}{H_t} \right]^{1-\theta} \frac{\theta - I_{t+1}/Y_{t+1}}{\theta - I_t/Y_t} = \frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta} \Sigma_{t+1}, \tag{12}$$

where $\eta_t = \left(1 - \delta_H + \frac{E_t}{H_t}\right)$, $\lambda_t = \left(1 - \delta_K + \frac{I_t}{K_t}\right)$ and $\Sigma_{t+1} = \frac{\theta - I_{t+1}/Y_{t+1}}{\theta - I_t/Y_t}$;

b) consumption and dividends share the same gross growth rate.

Proof The expression in a) is obtained by simple algebra, starting from the expression of dividends given above (see "Appendix 2"). The statement in b) implies that education expenses are part of the consumption. □

Given Remark 1 we can express the price to consumption ratio (PC) of the consumption claim $R_{c,t+1} = \left(\frac{P^c_{t+1} + C_{t+1}}{P^c_t} \right)$ as

$$PC_t^{\alpha} = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta} \Sigma_{t+1} \right)^{1-\gamma} (PC_{t+1} + 1)^{\alpha} \right]. \tag{13}$$

In the same fashion we can express the price to dividend ratio (PD) of the dividend claim[7] $R_{e,t+1} = \left(\frac{P^e_{t+1} + D_{t+1}}{P^e_t} \right)$ as

$$PD_t = \mathbb{E}_t \left[\beta^{\alpha} \left(\frac{A_{t+1}}{A_t} \lambda_t^{\theta} \eta_t^{1-\theta} \right)^{1-\gamma} \Sigma_{t+1} (PC_{t+1} + 1)^{\alpha-1} (PC_t)^{1-\alpha} (PD_{t+1} + 1) \right] \tag{14}$$

[7] "Appendix 3" provides the derivation for both Eqs. (13) and (14).

Following the approach of Lettau et al. (2008), we solve Eqs. 13 and 14 numerically.[8] Thus we can get the equity return from

$$R_{e,t+1} = \frac{1 + PD_{t+1}}{PD_t} \frac{D_{t+1}}{D_t}, \tag{15}$$

and calculate its second moment as

$$\mathbb{E}_t \left[R_{e,t+1}^2 \right] - \mathbb{E}_t \left[R_{e,t+1} \right]^2. \tag{16}$$

Finally, the risk-free rate can be expressed as $R_{f,t+1} = \left(\mathbb{E}_t \left[Q_{t+1} \right] \right)^{-1}$, from which we can calculate both its first and second moments.

3 Empirical analysis

3.1 Estimation

Having provided the theoretical implications given by our framework, we can now turn to the numerical analysis of the model.

The data used for calibration span from the beginning of the year 1952 to the end of 2011. The dataset is expressed in real per capita terms with a quarterly frequency and is seasonally adjusted. The financial series (prices and dividends) are on the S&P 500 composite, while the risk-free rate is the yield on the three-month Treasury bill. These series are from Robert J. Shiller's webpage[9] and the Federal Reserve Economic Data website.[10] The main economic series are downloaded from the website of Bureau of Economic Analysis.[11] Consumption is measured as quarterly real total personal consumption expenditure (NIPA table 2.3.6, line 1), GDP is quarterly real gross domestic income (NIPA table 1.1.6, line 1), investment in physical capital is quarterly non-residential fixed investment (NIPA table 5.3.5, line 2), and education expenditure is personal education and research expenditures (NIPA table 2.5.5, line 95). Both human capital and physical capital series are constructed using the perpetual inventory method. Finally, we use the official recession dates as reported on the website of the National Bureau of Economic Research.[12]

To estimate the technology shocks, a standard technique based on the growth accounting framework is applied. In particular, with constant return to scale, it is possible to decompose the output growth in two parts; thus the change in technology shocks can be estimated as

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[8] The problem is solved by fixed point iteration over the price-consumption ratio. Given the price-consumption ratio as a function of the state beliefs, we start from a first guess value and we calculate the value the price-consumption ratio at each point of the state beliefs grid. Next, we update the guess of the function using these new values and start over the iteration process.

[9] http://www.econ.yale.edu/~shiller/data.htm.

[10] http://research.stlouisfed.org/fred2/.

[11] http://www.bea.doc.gov/.

[12] http://www.nber.org/cycles.html/.

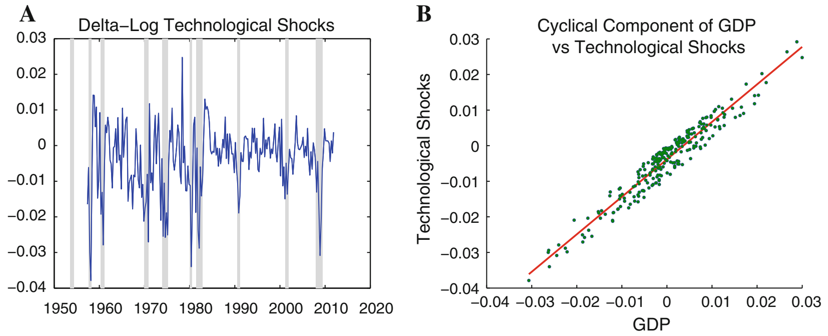

Fig. 1 Estimated technological shocks. This figure plots the empirical estimate of technological shocks. Data, transformed with logarithms, are quarterly starting from I-1952 to IV-2011. **A** Plots the estimated series of technological shocks, coupled with the recession periods according to NBER (*shadow areas* in the graph). **b** A scatter-plot of GDP cyclical component, estimated with a HP filter, versus the shocks

$$\Delta \log A = \Delta \log Y - \theta \Delta \log K - (1 - \theta)\, \Delta \log H. \quad (17)$$

Figure 1 shows the time series of the estimated technological shocks. A procyclical behavior in the series is clearly evident.

The regime-switching specification for the US economy with two possible states both for the mean and the volatility of the productivity shocks is also estimated. Parameter estimates for the model were computed using a Markov chain Monte–Carlo (MCMC) procedure following Kim and Nelson (1999).

The results from this analysis are given in Table 1. An important finding from our estimation is the high persistence of the states associated with the mean. In fact, the probabilities for the first moment of switching from the two states are 5.16 and 20.29 %, respectively. This implies an average duration of almost four years (19.5 quarters) for high mean states (associated with booms), and more than one year (5 quarters) for low mean states (associated with busts). Hence, if we find ourselves in either of the two states, we expect to stay there for several periods. The results are more striking for the second moment. Looking at its switching probabilities, it is clear how the volatility state is more persistent, so if we find ourselves in either one of the two states for the volatility, it is very well the case that we will face that state for almost a decade.

To get a final grasp on the estimation of the regime-switching economy, we investigate the capability of the model in picking up the historical business cycles of the US postwar economy. Figure 2 reports this analysis by plotting the estimated posterior probability, associated with the mean of the productivity shock process, of being in the recession state. It is evident how the Markov switching model is able to capture fairly well the US recessions as chronicled by the official NBER business cycle dates (the gray areas in the graph).

Figure 3 reports a similar analysis by plotting the estimated posterior probabilities, associated with the volatility of the productivity shock process, of being in the low state. The obtained graph is consistent with the declining macroeconomic volatility starting in the mid-eighties and documented widely in the literature (see Blanchard and Simon 2001; Lettau et al. 2008, among others), also named as "the great moderation"

Table 1 Estimation of regime switching economy

State	$\mu(s)$	$\sigma(s)$	p_{ij}^{μ}	p_{ij}^{σ}
Technological shocks' process estimation				
High (s = high)	−0.0011	0.0095	0.0516	0.0339
	(0.0008)	(0.0009)	(0.0203)	(0.0230)
Low (s = low)	−0.0168	0.0042	0.2029	0.0309
	(0.0082)	(0.0005)	(0.0681)	(0.0239)

This table reports the estimated parameters of a two-state Markov switching model for the US postwar economy. The estimates are based on the MCMC algorithm from Kim and Nelson (1999) with both the mean and the volatility of technological shocks being different in the two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2011; source: BEA). Standard errors are reported in parentheses

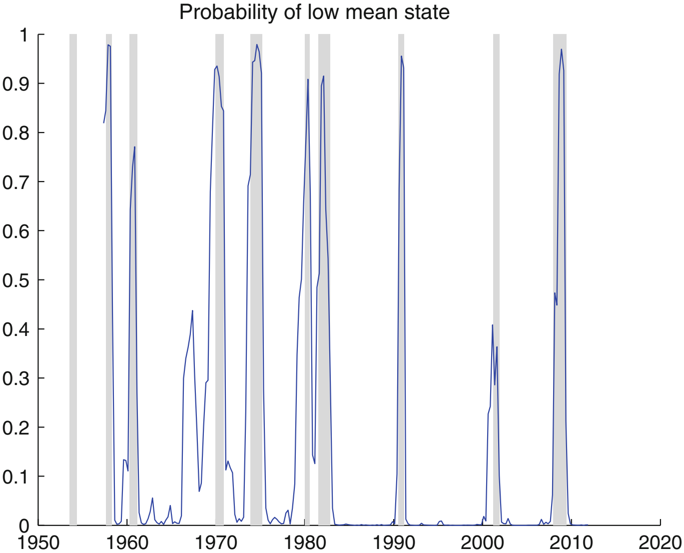

Fig. 2 Posterior probabilities of a recession. This figure shows the estimated posterior probabilities of being in a recession coupled with the official NBER recession dates (*shadow area*). Data employed in the estimation are quarterly starting from I-1952 to IV-2011

by Stock and Watson (2002). Moreover, the figure clearly shows how the increase in the macroeconomic risk due to the two most recent recessions is captured by modeling the volatility as regime-dependent.

3.2 Calibration

The basic calibration sets the model's parameters as follows: following Kydland and Prescott (1982) the share of physical capital (θ) is fixed to 0.36 and the depreciation rate for physical capital is fixed at 0.021. For human capital, we follow Heckman (1976) by choosing a value of 0.009.

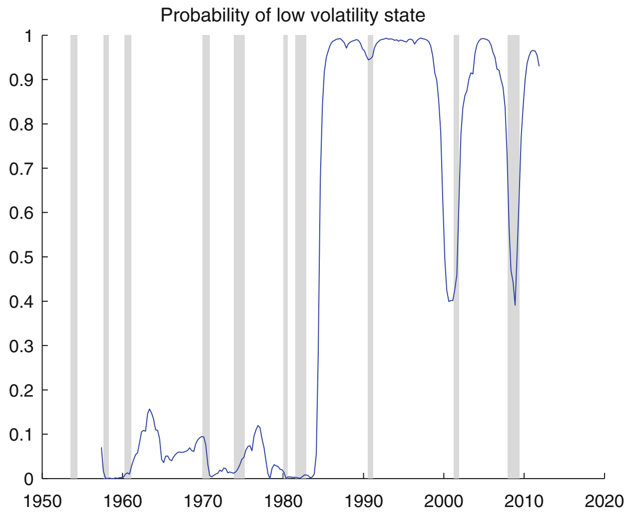

Fig. 3 Posterior probabilities of the low volatility state. This figure shows the estimated posterior probabilities of being in a low volatility state coupled with the official NBER recession dates (*shadow area*). Data employed in the estimation are quarterly starting from I-1952 to IV-2011

The ratio of investment to physical capital and the ratio of education expenses to human capital are set to replicate the long run mean of the Σ_{t+1} ratio obtained by simulation. This gives investment to capital ratios of 0.0475 and 0.0014 during booms and during busts respectively. Similarly, education expenses to human capital ratios are set to 0.02 and 0.0045 respectively.[13]

3.3 Macroeconomic implications

As a first step in our analysis, we investigate the capability of the model to match the macroeconomic figures for the US postwar data. The results reported in Table 2 are obtained by simulating 50 years of data based on the modeled economy. We replicate the simulation 500,000 times using the parameters of the two-state Markov switching model provided in Sect. 3.1. When compared with the US postwar data, the model overestimates the first moments of the yearly growth rates of consumption and output by around 100 basis points. Instead, while the implied correlation between consumption and output growth rates is in line with the data (cf. first line, panel 2 of Table 2), the model-implied volatilities are higher than what we observe in the US postwar economy. This bias is particularly strong for the consumption volatility that is 6 times larger than the observed figure. In the second panel, we report the correlation between output growth rates and asset returns. While the model-implied correlation between

[13] It is worth noting that we can use the average duration of the business cycles, estimated with the MCMC procedure, to calculate the long run average of the two ratios. By doing that, we obtain 0.0383 for the investment to capital ratio and 0.016 for the education expenses to human capital ratio. The two values turn out to be remarkably close to their empirical counterparts calculated on the US postwar data, which are equal to 0.037 and 0.018 respectively.

Table 2 Macroeconomic moments

Series	ΔGDP Data	ΔGDP Model	ΔC Data	ΔC Model
Average	0.019	0.028	0.021	0.027
Volatility	0.018	0.034	0.014	0.068
Correlation	Data	Model		
$\Delta GDP, \Delta C$	0.692	0.586		
$\Delta GDP, R_e$	0.325	0.249		
$\Delta GDP, R_f$	−0.001	0.586		
Series	I/Y Data	I/Y Model	E/C Data	E/C Model
Average	0.079	0.062	0.008	0.007
Volatility	0.020	0.049	0.002	0.003

This table reports the results obtained by simulating (500,000 times) 50 years of data based on the modeled economy. The simulations are using the parameters of the two-state Markov switching model provided in Sect. 3.1, where both the mean and the volatility of technological shocks are different in the two possible states of the economy. Estimations are compared with the US postwar data from 1957 to 2011

the equity claim and the output is in line with the correlation observed in the data, the correlation between the risk free claim and the output is largely overestimated by the model. Finally, in the third panel of Table 2, we report figures for the ratios of the investment in the two production factors with respect to output and consumption. On the one hand, the model produces an investment in the human capital ratio (E/C) that is in line with its empirical counterpart both in the first and in the second moments. On the other hand, the model implied ratio of production investment to output (I/Y), has the same average but is more volatile than the observed data.

The unsatisfactory performance in terms of the consumption volatility is not new in the literature (see Jermann 1998; Tallarini 2000, among others) and can be explained by the linear relation between dividends and consumption implied by the model.[14] In the same fashion, the direct link between consumption growth rates and the risk free asset explains the overestimated correlation between the latter and the output growth rates: risk free movements are completely driven by the consumption path which determines the risk free and output co-movements as well.

3.4 Asset pricing implications

In this subsection we present the asset pricing results obtained by solving the model numerically.

As a first comparison, we set the utility function parameters as in Lettau et al. (2008): $\gamma = 30$, $\psi = 1.5$, and we fix $\beta = 0.9925$.

[14] An important assumption in Lettau et al. (2008) is that dividends on equity are a power function of aggregate consumption, that is they assume leverage as in Abel (1999). This would be a straightforward way to reduce consumption volatility keeping the dividends risky enough to have a sizable equity premium

Table 3 Financial series of the US economy using Lettau et al. (2008) utility parameters

Series	Mean data	Mean model	Std. data	Std. model
Equity	0.064	0.083	0.076	0.037
Risk Free	0.011	−0.061	0.008	0.001
Equity premium	0.053	0.144	0.076	0.053
$\rho(r^f; r^e)$	0.079	−0.092	–	–
$\rho(r^e_{t+1}; r^e_t)$	0.145	0.602	–	–
$\rho(r^f_{t+1}; r^f_t)$	0.297	0.602	–	–

This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2011; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/~shiller/data.htm). The coefficient of risk aversion is set to 30, the EIS is set to 1.5

The basic results from this calibration are presented in Table 3. It reports the set of estimates discarding the first five years of data, in order to address the well-known critique to the perpetual inventory methodology used in the capital estimations.

As shown in this first set of results, the model overshoots in estimating the mean equity premium. This is mainly due to the poor performance in matching the risk-free rate. In fact, with the proposed parametrization, we obtain a real risk-free rate that is negative and big (−6.1 %).

Given the unsatisfactory performance of the previous calibration, especially regarding the risk free rate, we perform a simple exercise: we fix the risk aversion parameter to 10, and we let the elasticity of intertemporal substitution vary to match the mean risk free rate of the postwar US economy.[15] The results for this calibration, obtained discarding the first five years of data, are reported in Table 4.

As shown in this table, the model performs well in matching the mean equity premium and the first moment of the equity asset. In fact the model predicts a yearly equity return of 6.4 %, and we match the real risk-free return to a level of 1.1 %, with an EIS parameter of 1.31. This leads to a predicted yearly equity premium of 5.3 % that is in line with the data. The matching of second moments is also satisfactory. In particular, both the standard deviations of the risk-free asset and the equity asset are of the same magnitude as their empirical counterparts. Regarding the correlation and the autocorrelation of the assets, the model performs well in matching the correlation between equity and the risk-free asset. Less satisfaction is derived from the performance in matching the autocorrelation of the two assets. This is probably due to the

[15] It is well known that Mehra and Prescott (1985) indicate 10 as an upper bound for an acceptable RA parameter in their setting. But is is important to point out that even if a risk aversion parameter higher than 10 can be perceived as implausible in a standard power utility setting, the parameter's implications in a Epstein–Zin utility framework change dramatically with respect to the power utility case. For a detailed theoretical characterization of these implications see Campanale et al. (2009).

Table 4 Empirical series of US financial markets matching the risk free rate

Series	Mean data	Mean model	Std. data	Std. model
Equity	0.064	0.064	0.076	0.038
Risk Free	0.011	0.011	0.008	0.011
Equity premium	0.053	0.053	0.076	0.054
$\rho(r^f; r^e)$	0.079	0.072	–	–
$\rho(r^e_{t+1}; r^e_t)$	0.145	0.658	–	–
$\rho(r^f_{t+1}; r^f_t)$	0.297	0.669	–	–

This table shows the asset returns implied by the model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2011; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/~shiller/data.htm). The coefficient of risk aversion is set to 10. The obtained EIS by matching the empirical mean of the risk free rate is 1.31

nature of the model in which prices have a strong persistence with respect to the states of the economy.

The drastic improvement in the model performance is due to the decrease in risk aversion and the interplay between the elasticity of intertemporal substitution and the risk aversion itself. By lowering the agent risk aversion we directly influence the equity claim: a lower value of γ makes equities more appealing; this leads to higher prices and so decreases the equity return. Also, a lower risk aversion makes interest rates less sensitive to consumption growth while a lower EIS makes the agent more concerned about consumption swings between boom and recessions: the lower the EIS the lower the willingness of an agent to transfer her wealth overtime. The latter effect seems to prevail reducing the appeal of the risk-free claim and increasing the equilibrium interest rate.

Before moving ahead in our empirical analysis, it is worth analyzing the value obtained for the EIS parameter to match the risk-free return, given that the empirical estimates in the literature vary considerably. One approach of the empirical research focuses on a representative agent setup and uses aggregate consumption data. This leads to estimates of the EIS coefficient below 1, and even close to 0 (see e.g. Hall 1988; Campbell and Mankiw 1989, 1991; Hahm 1998; Yogo 2004; Zhang 2006). Another strand of research relies on microeconomic survey data to avoid potential biases in the aggregate data. By considering a stockholder these studies find EIS parameters around or above 1. (see Beaudry and Wincoop 1996; Vissing-Jørgensen 2002; Vissing-Jørgensen and Attanasio 2003; Guvenen 2006). Although there is a lack of consensus in the economic literature, the recent asset-pricing literature relies on the higher EIS estimates of the latter literature, both Bansal and Yaron (2004) or Lettau et al. (2008) calibrate their models with an EIS greater than one. This choice is mainly linked to the capability of generating procyclical prices when agents have a recursive utility function. Consequently, we can consider the value of 1.31 for the EIS, obtained by matching the risk-free rate, to be in line with the latter literature and theoretically well grounded.

Table 5 Estimated parameters for the restricted regime switching model

State	$\mu(s)$	σ	p_{ij}^{μ}
High (s = high)	−0.0007		0.0582
	(0.0007)	0.0071	(0.0208)
Low (l = low)	−0.0164	(0.0004)	0.2042
	(0.0018)		(0.0626)

Reported are the estimated parameters for the restricted version of the model. The estimates are based on a MCMC algorithm from Kim and Nelson (1999) with the mean of technology shocks being different in two possible states. The estimation is performed using real quarterly data (Q1:1952–Q4:2011; source: BEA). Standard errors are reported in parenthesis

3.5 The role of macroeconomic risk

Roughly speaking, the insight we can gather from Lettau et al. (2008) is that the reduction in macroeconomic volatility in the last twenty years can account for a good portion of asset valuation in the recent past. Thus, it is natural to inquire about the role of the volatility regimes in the economy that we are studying. We then re-estimate our regime switching economy, imposing the restriction of a single state for the volatility of the productivity shocks.

The parameters for the restricted regime switching model are obtained using the same Markov chain Monte–Carlo (MCMC) algorithm used for the unrestricted model. The resulting estimates are given in Table 5.

The estimated value for the volatility is in between the values we obtained for the two-state version of the regime-switching process. The main departure from the unrestricted estimation is that the difference between the means in the two states is sharper. Furthermore, the estimated probabilities of switching from the two mean states are 5.82 and 20.42 %, respectively. These probabilities confirm the persistence of each state also in the restricted setting.

We can now analyze the performance of the model where the decline in the macroeconomic risk is not considered. We re-estimate the main financial figures, implied by our model with a restricted regime-switching economy, fixing the utility parameters at $\gamma = 10$ and $\psi = 1.31$. The results for this calibration, obtained by discarding the first five years of data, are presented in Table 6. The implied equity returns are lower if we disregard the decline in the macroeconomic risk: we underestimated it by almost 30 basis points over the yearly estimate of 6.4 % obtained by modeling the macroeconomic volatility. The implied risk free rate is also effected and it moves in the expected direction. In fact we obtain an estimate of 1.0 % on an annual basis, that is about 10 basis point lower than the observed risk-free rate on which we calibrated the EIS parameter in the unrestricted version of the model (cf. Table 4). It is worth noting that the relatively small effects on both the implied risk-free rate and the implied equity return, are due to the reduction in the estimated probability of the low volatility state induced by the 2001 and the 2007–2009 recessions (cf. Fig. 3).

This result can be better interpreted if we focus on the role played by the volatility of the underlying state of the economy in determining the prices and thus the returns.

Table 6 Empirical series of US financial markets: restricted regime switching model

Series	Mean data	Mean model	Std. data	Std. model
Equity	0.064	0.061	0.076	0.040
Risk free	0.011	0.010	0.008	0.012
Equity premium	0.053	0.051	0.076	0.056
$\rho(r^f; r^e)$	0.079	0.079	–	–
$\rho(r^e_{t+1}; r^e_t)$	0.145	0.602	–	–
$\rho(r^f_{t+1}; r^f_t)$	0.297	0.602	–	–

This table shows the asset returns implied by the restricted model calibrated on the US postwar economy. The estimation is based on real quarterly data (Q1:1952–Q4:2011; source: BEA). Reported are the estimates obtained by calculating capital and education investments on the whole sample and discarding the first five years of data to address the critiques to the perpetual inventory methodology. The market dataset is from Professor Robert J. Shiller webpage (http://www.econ.yale.edu/~shiller/data.htm). The coefficient of risk aversion is set to 10, the EIS is set to 1.31

When we shut down the regime on the volatility, we are preventing the agent from entering a persistent "high risk" regime associated with a high volatility of the economy, but we are also depriving her of the possibility of entering a persistent "low risk" regime. This creates an economy where a smoother path for the consumption claim is not available. Such a path is appealing for the representative agent of this economy, and this would push the price of the assets down increasing their returns. So, an economy with a very persistent low volatility regime gives a higher incentive to the representative agent to use the risk free asset to transfer consumption overtime, pushing its prices up and lowering the risk free return. Instead, when the probability of switching between lower and higher volatility regimes is high enough, the possibility of facing a high risk regime pushes the prices up, thus decreasing the asset returns.

4 Conclusion

This paper deals with asset pricing implications of production economies. In particular, we propose a simple extension to a standard Real Business Cycle (RBC) model where the economy switches between booms and busts and consumers have a recursive utility.

Our first contribution is a detailed theoretical analysis of the equity premium's determinants in the proposed framework. Secondly, we show that a plausible parametrization of the model conveys financial figures that are in line with the empirical observations on the US postwar data. This is possible by having a model implied volatility of the consumption growth rate larger than its empirical counterpart. A detailed analysis on the relative contribution of prominent parameters of the model is also provided. This allows us to clarify the role of different choices on the utility function. In particular, we discuss the role of risk aversion and elasticity of intertemporal substitution in determining asset prices and thus, in determining the equity premium. Furthermore, we study the role of macroeconomic risk in the proposed economy, providing support for modeling a regime dependent macroeconomic risk.

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5 Appendix 1: The firm's problem

The firm maximizes its value by choosing the optimal investment, given the current capital stock, the level of human capital hired, and the technology level. Let Q_t denote the stochastic discount factor. The firm's problem can be written as

$$f(.) = \max_{\{I_t, K_{,t+1}, H_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} Q_t \left\{ (Y_t - W_t H_t - I_t) - z_t (K_{t+1} - (1 - \delta_K) K_t - I_t) \right\} \right]. \tag{18}$$

where z_t is the shadow price of the capital accumulation constraint and W_t is the price paid for human capital.

The first order conditions are

$$\frac{\partial f(.)}{\partial H_t} = (1 - \theta) \frac{Y_t}{H_t} - W_t; \tag{19}$$

$$\frac{\partial f(.)}{\partial I_t} = -1 + z_t; \tag{20}$$

$$\frac{\partial f(.)}{\partial K_{t+1}} = \mathbb{E}_t \left[Q_{t+1} \left\{ \theta \frac{Y_t + 1}{K_t + 1} + z_{t+1}(1 - \delta_K) \right\} \right] - z_t. \tag{21}$$

Equating Eqs. (19) and (20) to 0 and substituting out z in Eq. (21), gives

$$1 = \mathbb{E}_t \left[Q_{t+1} R^I_{t+1} \right], \tag{22}$$

where R^I_{t+1} indicates the return on investment and is given by $\left(\theta \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_K) \right)$

6 Appendix 2: Deriving dividend growth rate

We can start from the dividend equation

$$D_t = \theta Y_t - I_t,$$

divide both sides by Y_t:

$$\frac{D_t}{Y_t} = \theta - \frac{I_t}{Y_t},$$

and divide D_{t+1}/Y_{t+1} by D_t/Y_t:

$$\frac{D_{t+1}}{D_t} = \frac{Y_{t+1}}{Y_t}\frac{\theta - I_{t+1}/Y_{t+1}}{\theta - I_t/Y_t}.$$

7 Appendix 3: Deriving asset prices

We can rewrite Eq. 11 as

$$\begin{aligned} Q_{t+1} &= \beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}}\left(\frac{P^c_{t+1}+C_{t+1}}{P_t}\right)^{\alpha-1} \\ &= \beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}}\left(\frac{P^c_{t+1}}{C_{t+1}}+1\right)^{\alpha-1}\left(\frac{P^c_t}{C_t}\right)^{1-\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1} \\ &= \beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1}\left(\frac{P^c_{t+1}}{C_{t+1}}+1\right)^{\alpha-1}\left(\frac{P^c_t}{C_t}\right)^{1-\alpha} \end{aligned} \tag{23}$$

which gives us an expression for the stochastic discount factor as a function of consumption and price of its claim.[16]

First we price the consumption claim $R_{c,t+1} = \left(\frac{P^c_{t+1}+C_{t+1}}{P^c_t}\right)$. From this we have

$$\mathbb{E}_t\left[\beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1}\left(\frac{P^c_{t+1}}{C_{t+1}}+1\right)^{\alpha-1}\left(\frac{P^c_t}{C_t}\right)^{1-\alpha}\left(\frac{P^c_{t+1}+C_{t+1}}{P^c_t}\right)\right] = 1 \tag{24}$$

Define $PC_t = \frac{P^c_t}{C_t}$

$$PC^{\alpha}_t = \mathbb{E}_t\left[\beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha}(PC_{t+1}+1)^{\alpha}\right]. \tag{25}$$

[16] It is worth noting that Q_{t+1} can be further simplified:

$$\begin{aligned} Q_{t+1} &= \beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{\alpha(1-\frac{1}{\psi})-1}\left(PC_{t+1}+1\right)^{\alpha-1}(PC_t)^{1-\alpha} \\ &= \beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{1-\gamma}{1-1/\psi}(1-\frac{1}{\psi})-1}\left(PC_{t+1}+1\right)^{\alpha-1}(PC_t)^{1-\alpha} \\ &= \beta^{\alpha}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(PC_{t+1}+1\right)^{\alpha-1}(PC_t)^{1-\alpha} \end{aligned}$$

where PC indicates the price consumption ratio on the consumption claim.

Then, we solve for the dividend claim $R_{e,t+1} = \left(\frac{P^e_{t+1}+D_{t+1}}{P^e_t}\right)$:

$$\mathbb{E}_t\left[\beta^\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1}\left(\frac{P^c_{t+1}}{C_{t+1}}+1\right)^{\alpha-1}\left(\frac{P^c_t}{C_t}\right)^{1-\alpha}\left(\frac{P^e_{t+1}+D_{t+1}}{P^e_t}\right)\right]=1 \quad (26)$$

Define $PD_t = \frac{P^e_t}{D_t}$

$$PD_t=\mathbb{E}_t\left[\beta^\alpha\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\alpha}{\psi}+\alpha-1}(PC_{t+1}+1)^{\alpha-1}(PC_t)^{1-\alpha}(PD_{t+1}+1)\left(\frac{D_{t+1}}{D_t}\right)\right]. \quad (27)$$

Finally, using Remark 1, we can plug $\frac{A_{t+1}}{A_t}\lambda_t^\theta\eta_t^{1-\theta}\Sigma_{t+1}$ in place of the growth rates of consumption and dividends in Eqs. 25 and 27 to arrive at Eqs. 13 and 14 respectively.

We can solve this set of equations using the same technique as Lettau et al. (2008) and the estimation on the $\Delta \log A_{t+1}$ process. In other words, we solve for the fixed point and then compute expected PC and PD using posterior probabilities from the Hamilton (1989) filter.

References

Abel, A.B.: Exact solutions for expected rates of return under markov regime switching: implications for the equity premium puzzle. J Money Credit Bank **26**(3), 345–361 (1994)

Abel, A.B.: Risk premia and term premia in general equilibrium. J Monet Econ **43**(1), 3–33 (1999)

Ai, H.: Information quality and long run risk: asset pricing implications. J Financ **65**(4), 1333–1365 (2010)

Bansal, R., Yaron, A.: Risks for the long run: a potential resolution of asset pricing puzzles. J Financ **59**(4), 1481–1509 (2004)

Barro, R.J., Sala-I-Martin, X.: Economic Growth. The MIT Press, Cambridge (2004)

Beaudry, P., van Wincoop, E.: The intertemporal elasticity of substitution: an exploration using a us panel of state data. Economica **63**(251), 495–512 (1996)

Blanchard, O., Simon, J.: The long and large decline in U.S. output volatility. Brookings Papers Econ Activity **2001**(1), 135–164 (2001)

Boldrin, M., Christiano, L.J., Fisher, J.D.M.: Habit persistence, asset returns, and the business cycle. Am Econ Rev **91**(1), 149–166 (2001)

Brandt, M.W., Zeng, Q., Zhang, L.: Equilibrium stock return dynamics under alternative rules of learning about hidden states. J Econ Dyn Control **28**(10), 1925–1954 (2004)

Cagetti, M., Hansen, L.P., Sargent, T., Williams, N.: Robustness and pricing with uncertain growth. Rev Financ Stud **15**(2), 363–404 (2002)

Campanale, C., Castro, R., Clementi, G.L.: Asset pricing in a production economy with chew-dekel preferences. Rev Econ Dyn **13**(2), 379–402 (2009)

Campbell, J.Y., Mankiw, N.G.: Consumption, income, and interest rates: reinterpreting the time series evidence. In: Blanchard, O.J., Fischer, S. (eds.) National Bureau of Economic Research Macroeconomics Annual 1989, pp. 185–216. MIT Press, Cambridge (1989)

Campbell, J.Y., Mankiw, N.G.: The response of consumption to income: a cross-country investigation. Eur Econ Rev **35**(4), 723–756 (1991)

Campbell, J.Y., Viceira, L.M.: Who should buy long-term bonds? Am Econ Rev **91**(1), 99–127 (2001)

Campbell, J.Y., Chan, Y.L., Viceira, L.M.: A multivariate model of strategic asset allocation. J Financ Econ **67**(1), 41–80 (2003)

Cecchetti, S.G., Lam, P.S., Mark, N.C.: Mean reversion in equilibrium asset prices. Am Econ Rev **80**(3), 398–418 (1990)

Cecchetti, S.G., Lam, P.S., Mark, N.C.: The equity premium and the risk-free rate: matching the moments. J Monet Econ **31**(1), 21–46 (1993)
Cochrane, J.H.: Volatility tests and efficient markets. J Monet Econ **27**(3), 463–485 (1991)
Croce, M.: Long-run productivity risk: a new hope for production-based asset pricing? Working paper, UNC (2012)
Danthine, J.P., Donaldson, J.B., Mehra, R.: The equity premium and the allocation of income risk. J Econ Dyn Control **16**(3–4), 509–532 (1992)
Epstein, L.G., Zin, S.: Substitution, risk aversion, and the temporal behavior of consumption growth and asset returns i: a theoretical framework. Econometrica **57**(4), 937–969 (1989)
Epstein, L.G., Zin, S.: Substitution, risk aversion, and the temporal behavior of consumption growth and asset returns i: a theoretical framework. J Polit Econ **99**(2), 263–286 (1991)
Gomes, F., Michaelides, A.: Asset pricing with limited risk sharing and heterogeneous agents. Rev Financ Stud **21**(1), 415–448 (2008)
Guvenen, F.: Reconciling conflicting evidence on the elasticity of intertemporal substitution: a macroeconomic perspective. J Monet Econ **53**(7), 145–1472 (2006)
Hahm, J.H.: Consumption adjustment to real interest rates: intertemporal substitution revisited. J Econ Dyn Control **22**(2), 293–320 (1998)
Hall, R.E.: Intertemporal substitution in consumption. J Polit Econ **96**(2), 339–357 (1988)
Hamilton, J.D.: A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica **57**(2), 357–384 (1989)
Heckman, J.J.: A life-cycle model of earnings, learning, and consumption. J Polit Econ **84**(4), S11–44 (1976)
Jermann, U.J.: Asset pricing in production economies. J Monet Econ **41**(2), 257–275 (1998)
Kaltenbrunner, G., Lochstoer, L.A.: Long-run risk through consumption smoothing. Rev Financ Stud **23**(8), 3190–3224 (2010)
Kandel, S., Stambaugh, R.F.: Asset returns and intertemporal preferences. J Monet Econ **27**(1), 39–71 (1991)
Kim, C.J., Nelson, C.R.: State-Space Models with Regime Switching. MIT Press, Cambridge (1999)
Kydland, F.E., Prescott, E.C.: Time to build and aggregate fluctuations. Econometrica **50**(6), 1345–1370 (1982)
Lettau, M.: Inspecting the mechanism: close-form solutions for asset prices in real business cycle models. Econ J **113**(6), 550–575 (2003)
Lettau, M., Ludvigson, S.C., Wachter, J.A.: The declining equity premium: what role does macroeconomic risk play? Rev Financ Stud **21**(4), 1653–1687 (2008)
Malkhozov, A., Shamloo, M.: News Shocks, Volatility Shocks, and Asset Prices. Working paper , McGill University (2012)
Mehra, R., Prescott, E.C.: The equity premium: a puzzle. J Monet Econ **15**(2), 145–161 (1985)
Restoy, F., Rockinger, G.M.: On stock market returns and returns on investment. J Financ **49**(2), 543–556 (1994)
Rouwenhorst, K.G.: Asset pricing implications of equilibrium business cycle models. In: Cooley, T. (ed.) Frontiers of Business Cycle Research, Chap. 10. Princeton University Press, Princeton (1995)
Stock, J., Watson, M.W.: Has the Business Cycle Changed and Why? Working Paper 9127, NBER (2002)
Tallarini, T.D.: Risk-sensitive real business cycles. J Monet Econ **45**(3), 507–532 (2000)
Veronesi, P.: Stock market overreaction to bad news in good times: a rational expectations equilibrium model. Rev Financ Stud **12**(5), 975–1007 (1999)
Vissing-Jørgensen, A.: Limited asset market participation and the elasticity of intertemporal substitution. J Polit Econ **110**(4), 825–853 (2002)
Vissing-Jørgensen, A., Attanasio, O.P.: Stock-market participation, intertemporal substitution, and risk-aversion. Am Econ Rev **93**(2), 381–383 (2003)
Weil, P.: The equity premium puzzle and the risk free rate puzzle. J Monet Econ **24**(3), 401–421 (1989)
Whitelaw, R.F.: Stock market risk and return: an equilibrium approach. Rev Financ Stud **13**(2), 521–547 (2000)
Yogo, M.: Estimating the elasticity of intertemporal substitution when instruments are weak. Rev Econ Stat **86**(3), 797–810 (2004)
Zhang, Q.: Human capital, weak identification, and asset pricing. J Money Credit Bank **38**(4), 873–899 (2006)