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Why is the language of nature mathematical?

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> “La filosofia è scritta in questo grandissimo libro, che continuamente ci sta aperto innanzi agli occhi (io dico L’Universo), ma non si può intendere se prima non s’impara a intender la lingua, e conoscer i caratteri ne’ quali è scritto. Egli è scritto in lingua matematica, e i caratteri son triangoli, cerchi, ed altre figure geometriche, senza i quali mezzi è impossibile a intenderne umanamente parola; senza questi è un aggirarsi vanamente per un oscuro labirinto”
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> Galilei, *Il Saggiatore* 1623

§1 Introduction

This famous quotation by Galileo is important for various reasons. First of all, Galileo is creating a contrast between the *books made of papers* and the great Book of Nature that is constantly open in front of our eyes (the Universe). The books of papers of course are not always open, they need to be studied, but when we study them we discover that they often contain falsities, because most of these books, especially when they purport to describe nature, report the opinions and theories of other philosophers that have not studied nature with the right method. The Book of Nature (“the Universe, I say”) is instead constantly visible, constantly open in front of us, but if we look at it from the wrong perspective, that is, if we rely just on random observations, we will not learn much.

Secondly, *via* the alphabet of our natural language, composed by a few, simple symbols, we are capable of creating new expressions that have never been pronounced before as well as poems and works of literary art. Of course, the syntactic rules for combining the symbols are also essential. Analogously, the Book of Nature has a geometrical alphabet, and this needs to be learnt if we want to understand the way in which it has been written by God, otherwise “we would wonder vainly in a dark labyrinth”. The syntax of nature is established by the “sensate esperienze” (*sensible experiences*), since not all possible combination of geometrical shapes and algebraic symbols is effectively realized or exemplified by nature.

Galileo’s quotation is one of the first explicit, modern calls to arms for the mathematization of nature, a process that originated with the Pythagorean philosophers but that has continued till our days, and that is one of the main characteristics of contemporary physics. According to Galileo, the right method to study the Book of Nature, as Archimedes well knew, consists in combining “sensible experiences” with “necessary demonstrations” (*le certe dimostrazioni*). But while the 17th century mathematised nature in order to achieve a *mechanization* of our world picture, in the 20th century the *mathematization* of the physical world has reached unprecedented levels of sophistication. Theories like General Relativity, Quantum Mechanics, Quantum Field Theory, which have a fantastically accurate match with observations, could not be expressed precisely without the use of mathematics. Another giant of the history of physics expressed his credo this way: «Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas”»[1].

The following questions naturally arise: What does it mean to claim that mathematics (geometry in particular) is the “alphabet of nature”? Why is mathematics so “*unreasonably effective*” in the natural sciences? (Wigner 1967) (“Effective” here means *indispensible to predict and explain the physical world*). Galileo defended − sincerely or not it is not important to establish here − a *theological* answer to this question. According to Galileo, God, as in the platonic tradition of the Timaeus, was a geometer and created the universe after a geometrical pattern. The same creed was espoused by Copernicus and Kepler. However, is this the only explanation that we can offer for this process of mathematization of nature which involves not just physics, but also biology and the cognitive sciences?[2]

In the rest of this paper, I will try to sketch some possible answers to this crucial question.[3] In particular, in the next section (2) we will briefly discuss two pairs of opposite attitudes toward the problem of explaining why mathematics is so effective. In the third part (3) I will present the partial isomorphism solution, the one that I think is preferable.

§2 Two pairs of opposite attitudes toward the problem of explaining the effectiveness of mathematics

The first pair exemplifies what we could call a *mysteric vs a deflationary* attitude toward our problem. The former attitude is labelled *mysteric* because it amounts to saying that we will never be able to answer the questions raised in the introduction. Wigner himself is a representative of this "*Ignorabimus* attitude". As he wrote in his famous essay: "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve" (Wigner 1967, 232). The latter attitude is *deflationary*: Wigner's problem of explaining why mathematics is effective is *not* a genuine problem: thanks to a selective effect, that is, by focusing only on the evidence of success, and by ignoring evidence of *failure***,** one can make any hypothesis look good. The hypothesis that we are testing here is that the applicability of mathematics is a fact that deserves an explanation. As evidence of failure, one could think of the many instances in which mathematics cannot be applied to the physical world; since philosophers of mathematics have focussed so far only on the positive cases, they have thereby created a non-genuine problem. Furthermore, it is no mystery that mathematics is successful in physics, since it is physics that stimulates the solution of a problem by the use of mathematics, thereby also stimulating the growth of mathematics.

Even admitting the presence of a selective effect (we tend to illegitimately forget of the many branches of mathematics that have not found any application yet), it must be recognized that mathematical theories created with no applicative aims have proved indispensible for many empirical disciplines. It is this unexpectedness that cries out for an explanation, since it is analogous to *unexpected* predictions of physical models that had been created to solve *different* physical problems. Examples in this sense are not difficult to find: non-Euclidean geometries for general relativity, prime numbers for creating secure coding, group theory for particle physics, fiber bundles for gauge field theory, knot theory and algebraic topology for quantum gravity, etc.

The second pair of opposite attitudes can be summarized by the contrast between *antinaturalistic* explanations and *naturalistic* ones. Antinaturalism, in its most recent formulation, claims that theological explanations are well-supported by the miracle of the appropriateness of the language of mathematics to nature (Steiner 1998). I will assume without further ado that this is not an acceptable explanation, but that it is historically important for the origin of the notion of “law of nature”, which in fact depends on the faith in a orderly creator that imposes his rationality, expressed by the regularities of the laws, to its creature (nature), in the same sense in which human legislators impose their norms to cities. In Descartes, for instance, the warrant for the preservation of linear momentum (mV) in the physical universe is given by the fact that God is immutable.[4]

On the other hand, a naturalistic, non-supernatural type of explanation is the one advocated here, one that, therefore, does not treat our problem neither as an irresolvable mystery nor like a trivial side effect of a biased sample of historical cases. One could defend an *evolutionary* and *cognitive* explanation for the origin of our mathematical ability, as a form of adaptation to a reality that is, however, amenable to the application of mathematics. In this sense, such a naturalistic explanation may have to accept the applicability of mathematics as a *fact* requiring some sort of *ontological* hypothesis. In this metaphysical hypothesis, one ought simply to explain how we have come to terms, in our evolutionary past, with a certain ontic fact that exists independently of us. Accepting this fact involves a "realistic" attitude toward mathematics, meaning that "physical reality itself is mathematical", which is another way of saying that nature instantiates mathematical structures.

A *semantic* approach, quite compatible with an evolutionary explanation of our mathematical capacities, would insist that mathematics is effective in predicting and explaining the world around us for the same reason that our natural languages are effective in these tasks. Note, however, that in order to ensure a semantic, referring link between ordinary language and reality − whether mathematical or not − we need not assume that reality has a mathematical structure that is *mirrored* by our sentences or propositions. Analogously, one might want to resist the above ontological move, that suggests that physical reality is a realization of an abstract structure.

In any case, it is important to distinguish (i) an explanation of our acquiring certain mathematical capacities for numbering objects and for dealing with shapes, from (ii) an explanation of the reason why mathematics is effective. While these two explanatory tasks are both important, they seem quite different in nature, as they are actually logically independent of each other. While the former task does certainly need an evolutionary context, in which it is shown how other intelligent animals too are capable of performing minimal mathematical tasks in which they can tell whether predators are one or many, the applicability problem cannot escape, it seems to me, some sort of philosophical account of the relationship between mathematical models and reality. By philosophical account I mean that the simpler explanation for the applicability of mathematics *does* consist in holding that the physical world exemplifies abstract mathematical structure as a matter of fact, in the same sense in which three mountain tops and three marks of chalk on a blackboard are *concrete* instantiation of the *abstract* structure of a triangle. This kind of answer also justifies the former strategy of explaining the long adaptation of our brains to nature *via* evolutionary biology and psychology.

3 The Partial isomorphism approach to the applicability problem

While in my Dorato (2005) I presented, discussed and criticized the algorithmic explanation of the applicability of mathematics, here I will briefly expand on the solution alluded to before, and called, after French (1999, 2000), the *partial isomorphism* approach. This position is discussed in the context of, and compatible with, two distinct philosophical positions on the nature of things. There are philosophers of science who follow Poincaré in holding that instantiated mathematical structures are the only aspect of the world that we can *know* (Worrall 1989). These philosophers are called *epistemic structural realists*: continuity across scientific change is guaranteed by continuity of mathematical structure across different, successive paradigms. But the relata of these structures are, however, unknown, and may change across history of science. The second school, *ontic structural realism*, holds that mathematical structures are the only things that we can know because that's all there is. This is the ontic version of structural realism, originally defended by Eddington, and today, among others, by Ladyman (1998).

In both of these structural realists positions (epistemic and ontic), the world is regarded as partially isomorphic to mathematical structures, so that there is a sense in which the Book of nature is mathematical, as Galilei had already said in the quotation with which we opened this paper. The reason of "partially" is explained by the fact that *not* all physical relations and properties (constituting the physical structure) are mirrored by mathematical models (the abstract, mathematical structure). The latter are always abstraction from the richness of details of the physical world. Otherwise, our mathematics would be intractably complicated.

A simple example that was historically very important for our self-image will illustrate the main features of the view that regards the physical world as being partially isomorphic to mathematical models.[5] If we model the Earth as a flat disc, we can faithfully reproduce only a very small portion of it, yet in the range of a few square kilometres the model is appropriate: the Earth's radius is large enough for our planet to have a relatively small curvature C= C=1/r^2. Of course, we also need to forget or abstract from mountains and valleys and only then can the region in question be regarded as "flat". And yet the Earth can be assimilated, in small, flat regions, a flat round disk, a fact which explains why it has been so regarded in well before the Greek civilization.

For larger areas, the sphere is of course a much more appropriate model, as the Greek knew quite well from astronomical phenomena like lunar eclipses (the Earth between the Sun and the Moon casts a round shadow on the latter). But also the sphere is only a partially adequate model, since the Earth is a sphere only *approximately*. As we know after Newton, the Earth is flattened at the two poles, due to the centrifugal force that have been pushing the regions along the equator away from the axe of rotation. And even if we treat the Earth as a spheroid, the Earth is of course not flat on its surface as a spheroid ought to be. So the Earth is partially a disc, partially a sphere, partially a spheroid, *or partially isomorphic to all these abstract structures*, even though there is a progressive more correct approximation with each subsequent model. Were we to decide that a more complicated figure should be invoked to model it, we would have to come up with a different shape. Of course, for many of our purposes of description and explanation, treating the Earth like a sphere works quite well (say, if we want to calculate the diameter of our planet).

Be that as it may, I want to conclude this short essay by pointing out that Galileo's realism about mathematics is a reasonable position to hold. The Earth is a sphere, or exemplifies that abstract shape, independently of us human beings, independently of our concepts and of our language. Of course, we need concepts and language in order to attribute the Earth its shape, but the latter is a mathematical property of the Earth that is quite independent of any subjective epistemic instruments. We can therefore conclude that the Earth can be said to truly possess a geometrical structure. Why this is so is certainly a very difficult fact to explain, one that should make reference to the details of its formation from the Sun (rotational effects, and so on). In any case, this is a question that, in its turn, cannot be answered without bringing in more complicated mathematical structures, exactly as predicted by Galileo in 1623.

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[1] Einstein, A, *The world as I see it*, 1934.

[2] Think of evolutionary game theory, population genetics, or the use of neural networks in simulating human learning processes.

[3] For a more thorough treatment of this question, see Dorato (2005).

[4] “In the beginning, [in his omnipotence] he created matter, along with its motion and rest; and now, merely by his regular concurrence, he preserves the same amount of motion and rest in the material universe as he put there in the beginning […] For we understand that God’s perfection involves not only his being immutable in himself, but also his operating in a manner that is always utterly constant and immutable. […]” (Descartes 1985, part II, art. 36).