WHY ARE (MOST) LAWS OF NATURE MATHEMATICAL?

Mauro Dorato

> *Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas*
>
> —Einstein

In a frequently quoted but scarcely read paper, the Hungarian physicist Eugene Wigner rediscovered a question that had been implicitly posed for the first time by the *Transcendental Aesthetics* of the "Critique of Pure Reason". More precisely, rather than asking, in the typical style of Kant, "how is mathematics possible", Wigner was wondering how it could be so "unreasonably effective in the natural sciences" (Wigner, 1967).

The effectiveness in question refers to the numerous cases of mathematical theories, often developed without regard to their possible applications, that later have played an important and unexpected descriptive, explanatory and predictive role in physics and other natural sciences. A frequently given example is that of the conic sections, already known by the Greeks before Christ and used by Kepler many centuries after their discovery to describe the orbits of celestial bodies. Even more striking is the case of non-Euclidean geometries, applied by Einstein to describe how heavy matter bends the structure of spacetime in his general theory of relativity: the theory of curved, non-Euclidean spaces had already been built a century earlier by Gauss, Lobacevski and Riemann. A literary quotation addressing the role of complex numbers, due to the German writer Robert Musil, will conclude my necessarily short list of

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examples: "The strange fact is that with these imaginary or even impossible numbers one can anyway make perfectly real calculations which end in a concrete result". Ironically, at the time of *The Confusions of the Young Törless* (1906), from which this passage is taken, Musil could not be aware at the fact that the most successful theory of the atomic structure of matter – quantum mechanics – would have been using imaginary numbers to calculate the probability of measurements.

In this paper, I want to raise once again Wigner's question (to which I will be referring as 'WQ') *in order to shed light on the related issue of the nature of scientific laws*. Namely, my main purpose is to show that typical questions of the philosophical literature on laws, like

1 what laws are[1] and

2 how we come to know them,

can be fruitfully approached afresh if we pay due attention to *their mathematical character*. Note first of all that if we replied to WQ by saying that "nature itself is mathematical", we would trivialize the question only in appearance, since such a metaphysical answer should itself be explained: if, say, mathematics is a creation of ours, why are laws of nature itself mathematical? On the other hand, the fact that the laws of science are mathematical poses the question of the mathematizability of nature, which provides the clue for a correct understanding of WQ.

Considering that the idealized and simplified character of physical laws – on which many philosophers have insisted – might be simply due to the fact that such laws are mathematical and mathematical equations must be *tractable and amenable to solutions*, it is quite surprising that no one has tackled the philosophical problems of scientific laws from the perspective of Wigner's question. To be sure, a possible explanation of this neglect could come from the immediate remark that *not all laws are mathematical or expressible in a quantitative language* ("metals tend to expand if heated" or "all ravens are black" are but two examples). However, most if not all physical laws are expressed mathematically, and mathematical models have become increasingly important in biology (think of evolutionary game theory), in the cognitive domain (neural networks), and in the social sciences (decision theory and game theory).

[1]Recall that, roughly, ontological realists about laws claim that statements expressing laws of nature refer to mind-independent properties and relations obtaining between natural systems, while semantic realists claim that law statements are approximately true, or at least are susceptible of receiving a definite truth-value. Ontological antirealists about laws deny them any referential character, while semantic antirealists regard law-statement as merely useful, but do not grant them the property of being true or false.

This fact justify an approach to laws that focuses on attempts at answering WQ.

In the first section of the paper (1), I will illustrate the significance of WQ for the issue of natural laws, as well as for the philosophy of mathematics and language, by putting the problem in context and by trying to argue that the problems it raises are *genuine*. This preliminary task is necessary, since if WQ were a pseudo-problem, the claim that it could help us to consider the issue of scientific laws in a new way would be groundless. In the second and third sections (2-3), I will present an important, current attempt at answering WQ, centered on the view that laws of nature are the *software* of the physical universe. In this metaphor, which for its proponents is suggestive of a deeper truth about laws, the universe is considered to be a gigantic computer whose *hardware* is whatever fundamental physics tells us about the ultimate component of matter (fields, particles, superstrings, etc) and whose software is the ordered sequence of states it goes through in time. In the fourth and fifth sections (4-5), I will critically discuss this approach to natural laws by raising various difficulties, some of which appear to be fatal. Finally (6), I will propose my own way to relate WQ to the issue of the nature of scientific laws, which recommends to look more closely at the complex operations of *measurement*. While as a solution to WQ this proposal is only a suggestion to look in a new direction, it helps us to see why mathematical laws represent the world in the same way (or very analogously to the way) in which a quantitative order introduced with a scale represent the relations between the properties of the systems to which we apply it.

1. Is Wigner's problem genuine? A puzzle in the philosophy of math and language

First of all, let us begin with the skeptical or even cynical remark that for every philosopher that regards a problem as fundamental and ineluctable, there is at least another one that claims that the same problem is not genuine. In our case, someone that might want to diminish or even dissolve the sense of mystery of the unexpected applicability of various parts of abstract mathematics to the description of the natural world, might want to point out that, first of all, *important branches of mathematics were explicitly developed to solve physical problems.* The case of *calculus* in Newton's *Principia* is an important example of a piece of mathematics that can be used to describe the relationship between certain physical magnitudes simply because it was devised for that purpose. Secondly, one could remark that *many parts of mathematics* *have no application whatsoever*, and Wigner's problem could simply be the effect of selection: for few pieces of mathematics that are applicable many more simply aren't, but we obviously take notice only of the former and not of the latter.

As a reply to the first objection above, consider that even if the motivation to create calculus or the theory of probability has come from empirical problems (physical questions of instantaneous speed or "less intellectual", combinatorial questions originated with dice tossing, respectively), this historical fact does not diminish at all the sense of mystery created by WQ. Let us suppose that we can divide all successful applications of mathematics into two classes, the expected and the unexpected ones. Though the latter make a greater impression upon us, and are therefore favored in the illustration of WQ for obvious rhetorical reasons, it is plausible to claim that also the former, sooner or later, will generate the same type of wonder of the unexpected ones.

For instance, note that once launched on its path after the applicative input, calculus *proceeded independently of empirical motivations or intended applications.* If it must be acknowledged that *advanced* calculus has applications to physics that had not been intended at all by the mathematicians that built on Newton and Leibniz, it then remains true that mathematical theories "give us in return much more than we originally put into them".

As a way to illustrate what the latter metaphor mean, consider that the relationship between mathematics and the empirical world resemble in this respect the relationship between the *theoretical* and the *observational* terms of a theory, as the mature Hempel understood it (Hempel, 1958). Mathematical theories, exactly like the theoretical terms of a theory, have an *open texture*, in the sense that the range of applicability of the most fruitful of them cannot be exhausted by the original, intended application for which they had been devised. In the same sense, the original operational definitions with which some theoretical term has been introduced does not give a translation of the latter, if the term is really fruitful.

In a word, the heart of the problem of the applicability, it seems, is that mathematics proceeds with a demonstrative method that is *a priori.* Physics and the other natural sciences are instead based on experiments, and the representative component of physical laws or models is certainly derived from our observations, and is therefore *a posteriori.* How is it possible that mathematical theorems, arrived at *via* rigorous, *a priori* derivations, can help us to discover properties of the natural world, which we can know only with the help of our observations, and therefore *a posteriori*? I think that this way of formulating Wigner's question –

which possibly was in the back of Kant's mind when he thought of mathematics as based on *synthetic a priori judgments* – clearly disposes of the skeptic's first objection.

As to the second one, based on the selective character of the examples usually given to illustrate WQ, consider that even if the case of the non-Euclidean geometries were the *only one* in favor of the significance of WQ, it would be so striking as to deserve an explanation. The fact is that it is difficult to think of a major area of mathematics that is really "immune" from applications (besides differential geometry, think of algebra and algebraic topology for particle physics). Furthermore, "pure" mathematicians trying to insulate their discipline from possible applications are either disappointed by some unexpected, later match with the empirical world, or do not serve their discipline in the best possible way.[2]

Taking now for granted that WQ is genuine, let us note in passing that it should be discussed much more often than it is also in order to evaluate various philosophical positions on the nature of mathematical knowledge. *Prima facie*, it creates problem to many of such positions, and it is perhaps for this reason that it is so neglected.

Platonism is notoriously affected by the problem of explaining how it is that we can know, and therefore *causally interact with, a world of abstract entities which we discover*, and that has therefore not been created by our language or minds. Remembering that *abstract entities are causally inert by definition*, this objection clearly holds, provided that knowing x presupposes to some extent that we must causally interact with x. However, even if we could circumvent this problem by denying the premise of the causal theory of knowledge, we would have to explain why the *physical* world, which is spatio-temporally extended, should mirror the structure of the *mathematical* world, in such a way as to allow the application of concepts inhabiting the latter to the former.

Within *constructivist* positions, we must ask why a *creation* of ours can carry so much descriptive and predictive power, enabling us to explain and systematize entities of the natural world which we obviously did *not* create. *Prima facie, it would seem that unless mathematical notions derive from our experience, it is difficult to make sense of the applicability of mathematics.* However, even if certain mathematical structures were the evolutionary product of a long process of cognitive adap-

[2]As von Neumann once wrote, separating mathematics from its empirical applications can only transform it into a sterile intellectual game. This of course does not mean that mathematicians should not continue to pursue their work without having some application in mind, since it is impossible to predict which mathematical branch will be susceptible of being more fruitful.

tation of our brain to objects whose size is comparable to our bodies, we would have to explain why these structure have been successful also in describing entities whose size is much smaller and much bigger than our bodies by various orders of magnitudes (think of the application of group theory in particle physics).

One further attempt at explaining WQ that might sound slightly deflationary comes from the claim that mathematics are "effective" for the same reasons why our natural languages are.[3] After all, mathematics is a particular type of language, only more abstract and semantically more precise than ordinary languages, and its applicability to the external world should generate no more surprise than the fact that we use Italian, English or Danish to refer to the world around us (we "apply" them).

Furthermore, since *mathematical abilities* are to a good extent genetically determined, fundamental mathematical concepts, like *number* or *space*, might be innate, in the same sense in which fundamental concepts are innate in Fodor's *language of thought*: otherwise, we might ask, what would be the *object* of such mathematical abilities? If we suppose, in addition, that the contents of our thoughts are expressed in symbolic structures of an innate language, whose syntax and semantics are similar to (though more abstract than) those of the natural language, *then* the claim that all our mental processes are *computational* would explain the *deductive* character of mathematics and would also explain why mathematical knowledge is *a priori* without invoking any outlandish form of Platonism.

The connection between WQ and the philosophy of language is undeniable, and would deserve much more empirical and conceptual attention than it usually does. However, here I feel justified in simply mentioning it, since I surmise that a further exploration at this stage of our knowledge of computational knowledge would not help us to come any nearer an understanding of the nature of scientific laws.

In any case, also the prospect of a linguistic approach *vis à vis* the applicability question might raise some reservations. For one thing, the mathematical method of proving, that is, of ascertaining the justifiability of an assertion *via* deductions, is unparalleled in "natural" uses of natural languages. Of course, we can use the rules of logic to deduce an assertion from other assertions, all cast in a natural language. However, this reply has little force, to the extent that the use in question is already part of a regimented way of conducing our intellect, *logic*, which is itself

[3]I owe this suggestion to Jan Faye, with whom I have had several conversations on this problem.

a branch of mathematics: in this sense the reply is part of the problem we are trying to clarify, which is why mathematics is effective. What is more important, the *epistemology* of natural language and of mathematics *seem* very different, insofar as the latter makes use of *a priori* knowledge in a much more systematic and extended way, while within the former the (referential and causal) contact with the empirical world apparently plays a much more important role (*a posteriori* knowledge).

2. Three ingredients to make mathematical laws

Our species is characterized first and above all by its ability to build artifacts. As a consequence, in the attempt to try to understand the unfamiliar and the unknown in terms of the known and the familiar, *the latter has often been equated with what we can build, for the simple reason that we know how it works.* Given the immense importance that computers have in our society, it can come scarcely as a surprise that physicists and philosophers have relied on computer science not only in order to understand the nature of laws, but also in trying to give a tentative answer to Wigner's question about the applicability of mathematics.

Essentially, the idea is that we can look at any physical system from two viewpoints, an "anatomic" one, which means that we look at its components (corresponding to the *hardware* of a computer) and a physiological, functional one, which means that we look at what the system does, or the laws controlling its behavior (corresponding to the software of a computer). If the universe can be regarded as one unique, gigantic physical system, the laws that govern its unfolding in time could be regarded as *the software of the universe*. In order to further explore this metaphor and understand its significance for the question of laws, consider that, following Whewell, there are three components that are necessary to express a law of nature in mathematical language:

1 the initial or boundary conditions, i.e., *the inputs*, or the numerical sequence that we obtain *via* a measurement;

2 the *algorithmic structure*, given by the mathematical formula that we apply to the data in 1;

3 the quantities that are left invariant by the application of 2 to 1, namely the *constants of nature*.[4]

[4]For this tripartite structure, see also Barrow, 1988, p. 279.

Whewell referred to 1 as the determination of the independent variable; to 2 as the discovery of the formula that is capable of "colligating" the independent to the dependent variable ("the colligation of facts"), and to 3 as the determination of the coefficients (see Butts, 1968, pp. 210, 211).

I think it is safe to claim that a philosophical discussion on the nature of scientific laws that did not take these factors into account would not be complete, and would run the risk of separating the philosophical analysis from the scientific practice and from what scientists usually mean by "scientific law". Even though not all scientific laws are mathematical, it is not implausible to suppose that many neo-positivist accounts of laws suffered from the original sin of supposing that universal, but "*qualitative*" statements of the kind "all ravens are black", could be considered paradigmatic examples of natural laws.[5] Such statements do not contain any of the three ingredients mentioned above, and it is perhaps for this reason that a good part of the philosophical debate on laws – think of the failed attempts to separate genuine laws from universal generalizations on a purely syntactic level – has often been so remote from the scientific practice to become a purely academic game.

I do not think that I am endorsing here a form of mathematical chauvinism. Clearly, since the mathematization of laws is a phenomenon that prevails in particular within physics and some branches of biology and economics, it is clear that it is in these empirical sciences that the problem of why laws are mathematical apply. However, it is not difficult to show that what is philosophically relevant about the way in which mathematically expressed law represent the world can be extended without too many difficulties to the laws of other, less mathematized sciences. By quoting Kant, and thereby using for once the principle of authority, we can consider the more mathematized sciences to be a model of objective knowledge from which the other sciences draw an inspiration: "Since in any theory about nature one can find so much science, properly speaking, as there is knowledge *a priori*, it follows that the doctrine of nature can contain so much science, properly speaking, as there is mathematics that can be applied to it." (Kant, 1968, p. 470 – my translation)

3. Laws as the software of physical systems

Focusing just on the first two components of the three points presented above (the third is not relevant for our purposes), we can regard

[5]Here "qualitative" refers to the purely classificatory nature of the predicates (in the example, black) appearing in the universally quantified statements.

a mathematically formulated law as a "bridge" colligating two banks of a river, each constituted by quantitative data resulting from measurements. On one side of the river we find the initial data or boundary conditions – 1 above – which in our metaphor we can regard as the input, and on the other side we find the predictions or retrodictions – the *output* – the result of a *calculus.*

Since such a result is obtained in a purely deductive fashion, that is, thanks to the application of the law to the initial data, the metaphor of the scientific laws regarded as the algorithm of a computer appears initially justified. If the initial data in fact are such as to satisfy some mathematical conditions which in this context can be omitted,[6] and whenever the solution to the equation exists and is unique, a mathematical law expressed as a differential equation enables us to transform in a finite numbers of steps, and in purely mechanical fashion, the initial data in final predictions (output).

What interests us is, of course, whether such an analogy between the laws of succession of any physical system – regarded as something that evolves in time by going through a finite number of states describable in physical language – and the software of a computer, can help us to better understand: (i) why the world is describable by mathematical laws and (ii) how the latter are related to the world, that is, how they *represent* it. In order to shed light on the presuppositions of the law-software analogy, we should ask whether also a physical system, in a sense to be specified, can be said to "compute" its "next" state *by causing it.* Can we say that a physical system going from an initial to a final state *literally executes a program or calculates its future state*, in the same sense in which a mathematical physicist deduces or calculates the predictions corresponding to the initial data?

On the basis of a physical version of Turing-Church hypothesis (according to which every intuitively computable function can be computed by a Turing machine), David Deutsch has recently tried to answer these questions by claiming that every physical process that can be performed in a finite number of steps can be *simulated* by a *quantum Turing machine* ("quantum" is needed because energy levels of finite systems must be, unlike classical ones, *discrete*) (Deutsch, 1985). If Deutsch's hypothesis proved to be correct, the relationship between a physical system undergoing a finite number of successive stages and a quantum Turing machine would be one of *simulation.* In this respect, the analogy between any physical system undergoing a temporal evolution in finite time and

[6]The functions representing the data must be differentiable at least as many times as the degree of the differential equation giving the algorithm.

the mathematical physicist performing a calculation would be given by the Turing machine acting as a "mediator": the capacities of both a natural system an of a human calculator (the physicist) to perform a series of operations in succession can both be simulated by a Turing machine, *if* the Turing machine hypothesis about the human mind holds water.

Suppose that the temporal evolution of any physical system is describable by *finite strings of real numbers*, corresponding to operational measures of physical magnitudes (temperature, pressure etc.). We can have two cases: such string can be ordered

$$(111000111000111000\ldots)$$

or truly random

$$(0100110101100110\ldots)$$

In the first case, the string can be generated by a simple instruction ("print 111000 n times"), which is much shorter than the list itself. In the second case, the string *appears* as truly random, where "appear" is meant to stress that while we can show that a finite string is not random by giving the generating law (algorithm), we can never prove that a string is random because of rigorous results in classical logic (the halting problem).

At this point we can give two definitions, based on algorithmic complexity theory, which will be relevant for our purpose:

DEF 1 *the complexity of a string is the length of the shortest algorithm capable of generating it,*

DEF 2 *a string is algorithmically compressible when there is an algorithm capable of generating it, such that its information content (number of bits) is much less than that of the string.*

As an illustration of these definitions, consider that a string like

$$\{1, 4, 9, 16, 25, 36, \ldots\} \tag{1}$$

is obviously not random, since it can be obtained by squaring the positive integers in the list

$$\{1, 2, 3, 4, 5, 6, \ldots\} \tag{2}$$

If the numbers in 2 correspond to measured magnitudes in such a way that, say in a temporal interval of 1, 2, 3 seconds (the input data), a body travels 1, 4, 9 meters (the output), then the existence of a rule generating 1 from 2 shows that 1 is *algorithmically compressible*. The above algorithm is – *modulo* the constant $1/2g$. – Galileo's law of free fall, generating the spatial intervals 1 from the square of the temporal intervals 2.

In a word, by following the metaphor of scientific laws regarded as the software of a physical system, we discover that *searching for laws is tantamount to asking which is the length of the shortest program capable of generating the string of numbers expressing the experimental measures*. Such a length – the complexity of the string – will be equal to that of the original string only if the latter is composed by apparently random numbers, and does not obey any known law.

The idea that scientific laws are an economic synthesis of all the information contained in our observations is certainly not new, and in this algorithmic approach it finds a new, rigorous and precise formulation. It was especially Ernst Mach who regarded science and its theories and laws as a summary of our observations. As he wrote: "Science is a form of business. Its purpose is to find the maximum amount of the infinite eternal truth with the minimum amount of work, in the minimum expenditure of time and with the minimum amount of thought effort".

After having made explicit the philosophical consequences that seem to follow from the software metaphor for scientific laws, we can now finally discuss a possible explanation of the applicability of mathematics, due to the physicist John Barrow: "science exists because the natural world seems algorithmically compressible. The mathematical formulae that we call laws of nature are economical reductions of enormous sequences of data expressing changes of state of the world: here is what we mean by intelligibility of the world ... *Since the physical world is algorithmically compressible*, mathematics is useful to describe it because it is the language of the abbreviation of sequences. The human mind enables us to make contact with that world because our brain has the ability of compressing complex sequences of sense data in shorter form. Such abbreviations make thought and memory possible. The natural limits that nature poses to our senses prevent us from overloading our brains with information about the world. Such limits are security gates for our minds" (Barrow, 1992, pp. 93–96).

4. Does the software metaphor really work?

Let us now go into discussing the various difficulties of this interesting proposal, both as an answer to WQ and as a response to the issue of natural laws. If laws are nothing but *compressed observational information*, let us remark at the outset that the focus of this approach on laws is clearly *epistemic*. This seems to be confirmed by what Barrow claims about the brain, regarded as the main filter of information coming from sense data.

Unfortunately, claiming that laws are merely a more concise rendering of observational information seems to trap this view into the Procrustean bed of the early neo-positivist conception of theories, and of theoretical terms in particular. First of all, how can we explain the fact that known scientific laws can be often used to explain and predict wholly new phenomena if laws are – exactly like the theoretical terms were in the early neo-positivistic conception – a mere translation, that is, a shorter, more compact version, of observational statements entailed by measurements? Secondly, how can we account for the fact that some laws are purely theoretical and, as such, make no reference whatsoever to directly observable phenomena like phenomenological laws do?

Finally, how can we make sense of the *abstraction* and the *idealization* implicit in the construction of phenomenological and theoretical laws within the algorithmic conception of laws? In the actual world we certainly observe frictional effects, but the law of free fall and that of the isocrony of the pendulum *abstract* from them. From this viewpoint, *rather than passively reflect and summarize our observations*, physical laws select and "caricaturize" the property of a physical system.

Perhaps the defenders of the algorithmic approach to scientific laws can reply to each of these objections by pointing out that the *form* that a law takes in one field may have an heuristic role. For instance, it may suggest interesting *syntactical analogies* with the relations exemplified by data coming from unrelated fields, in the same sense in which Coulomb's law in electrostatics is syntactically related to Newton's law in mechanics. Even supposing that formal analogies of this sort are able to explain the *open character* and the fruitfulness of good theoretical laws, i.e., their being not confined to the data for which they had been devised, another question remains unanswered: why do formal analogies work? To this question the software approach to laws does not seem to be able to provide any satisfactory answer, except the one that consists in pointing to the repeatability of some patterns in the world as a brute fact of nature. On the epistemic side, it is easier to point out that we try to make the most of the symbols we have already successfully applied, and the more observations fall under a given mathematical structure, the simpler, the more economical, and more unified is our explanation of the world.

To come now to the second objection, involving theoretical laws that do not mention any directly observational term, we can always decide to make the observational-theoretical distinction a matter of degree, or to abandon the dubious notion of "direct observation". Don't physicists claim to observe, albeit indirectly, atoms and electrons? In this way, one can maintain that it is not just phenomenological laws that can be captured by the software metaphor, but also those theoretical laws regulating the behavior of entities that are only indirectly observable (atoms, electrons etc.).

Finally, in order to give due attention to the constructive, and selective character of physical laws (the third objection) it can be observed that the abstraction and idealization typical of physical laws might take place *before* their formulation, thanks to *a judicious choice of which quantities must be measured in order to find a formula colligating them.* This reply seems objectively the weakest of all, given that the law of inertia for example does not seem to depend on any choice of which quantities to measure.

The difficulties this position has in explaining WQ are even more serious, since it not clear what the relationship should be between the above mentioned capacity of the brain of filtering information in perceiving the world and the applicability of mathematics, even when the latter is regarded as the art of compressing sequences. Even the more ontological suggestion of Barrow's quotation does not help much, at least until we have a better grasp of what it means to claim that "the physical world is algorithmically compressible". Isn't this another way of formulating what we are trying to explain, namely the link between the regularity and the repeatability of certain phenomena of the world and our use of mathematics? An attribution "to the physical world in itself" of the compressibility in question seems to imply an acceptance of a classical empiricist, regularist position, according to which the objective content of any law is simply a spatiotemporally valid regularity. It is the repeatability of the phenomena subsumed under a law that is responsible for "the *redundancy* of the world", enabling us to compress the information coded in our observations.

A further difficulty of the information-theoretic approach to laws lies in the way in which the view is formulated: if it does not make sense to claim that a physical system literally and really computes, a "Turing test" for physical systems (involving a physical system regarded as a black box and a Turing machine observationally indistinguishable from it) becomes meaningless.

However, in response to this objection, consider that no physical systems, not even a computer, really "calculates" or "computes", if by such terms we refer to an *intentional, conscious act* accompanying a goal-oriented activity. Computers executing a program just undergo physical changes of states that *we interpret* as a computation, on the basis of a task that we have the machine perform for us. Certainly, physical laws must be such as to enable us to use physical processes to perform addition and multiplication, but any physical process, like our heartbeat or the motion of the Earth, can be used as a measuring and therefore as a calculating device. Clearly, such a use presupposes an intentional act of attribution of a function.

In this regard, to claim that a natural system *computes* its future state is no more metaphorical than saying that a computer calculates, given that both statements presuppose a function that we attribute to an inanimate object. Consequently, it is not possible to attack the identification between natural laws and algorithms executed by computers on the basis of the fact that only the latter *literally compute*. In fact, *either physical systems and computers both compute or they both don't*. And since we are inclined to call the machine I am using right now to write "computer", there is a sense in which we can extend this label also to more general physical systems.

5. Two fatal objections to the information-theoretic approach to laws of nature

If the objections we had examined so far can perhaps be rebutted, the two that I am about to present seem fatal to the whole algorithmic view of laws. The first simply points out that not all physical laws relate states ordered by the relationship of temporal succession "later than" (*laws of succession*). Many of them constrain physical states S existing *at the same time*, in such a way that no two such S's can be connected by causal or luminous signals (*laws of coexistence*).

The trouble might have already been anticipated by the reader. (i) If laws of coexistence link *spacelike-related* properties of physical systems; (ii) if *the notion of algorithm is essential sequential and temporal* (even in parallel computations, the results of distinct calculations must interact before the output); (iii) if laws refer to something existing mind-independently, then from (i) (ii) and (iii) it follows that *either all laws of coexistence can be reduced to laws of succession, or natural laws in general cannot be assimilated to computations or algorithms*.

Consider laws of coexistence like Newton's law of gravitational attraction, Gauss' law relating the electric flux through a close surface Φ generated by an electric charge q located within the surface, and Boyle's law relating pressure, volume and temperature:

$$F = G(M_1 M_2)/r^2$$

$$\Phi = \varepsilon_0 q$$

$$PV = kT$$

Except the first, which is an approximation to the more fundamental field equation of general relativity, these laws don't assume instantaneous action at a distance, so they are not in conflict with relativity or field theories, and neither can they be *prima facie* reduced to causal laws. Moreover, *despite the fact that they enable us to deduce or calculate one magnitude from the others* (say F or Φ or PV can be calculated from the magnitudes on the right-hand side of the equality sign of the equations), we cannot interpret them in an ontic way, namely *as if the system could really calculate one state from the one that is related to it in the mathematical formula.* In a word, the sequential character of laws is a necessary requirement for the view that laws are algorithms, given that the latter notion is intrinsically temporal: how could a Turing machine simulate the behavior of a physical system that is described by laws that merely specify the correlation of values in a spacelike hypersurface?

Clearly, if one wants to defend Deutsch's thesis, one must claim that all laws of coexistence are somehow less fundamental than laws of succession and depend (supervene) on the latter in some sense. Can we claim that all laws of coexistence *supervene* on laws of succession, in the sense that the origin of a law of coexistence presupposes a law o succession in the same sense in which a correlation between spacelike-related events *normally* presupposes a common cause in their past? If this supervenience thesis proved correct, the main obstacle against an attempt at giving an ontological interpretation of the algorithmic view of laws would be removed, because the existence of a law of coexistence would always presuppose a law of succession.

Unfortunately for this suggestion, any attempt at showing that laws of succession are more fundamental than laws of coexistence in the sense given above must face the existence or quantum correlations at a superluminal distance. For many philosophers, such correlations exclude the possibility to invoke a common cause in the past to explain the spacelike-related correlations among measurement results.[7] However, following Nancy Cartwright, we could regard the interaction of the two particles in the past *as a probabilistic common cause*, despite the lack of screening-off feature or of factorizability (Cartwright, 1994). But even if we managed to show that all quantum laws of coexistence originated from common causes and therefore from causal laws, this would not be equivalent to say that the laws of coexistence don't exist. And their mere existence is sufficient to cast serious doubts on the whole information-

[7]See for example van Fraassen, 1982.

theoretic approach to laws, unless, of course, its defenders were content with a mere epistemic interpretation. While there is nothing wrong in principle in limiting the impact of the algorithmic view to the epistemic side of the debate on laws, this self-imposed restriction seem to deplete the view of its significance. The algorithmic view of laws is interesting when it carries ontic implications: if it is just a way of rephrasing the well-known fact that we use differential equations to calculate the outcome of our measurements one might as well abandon it.

The second difficulty of the algorithmic view is the existence of non-computable equations in current physical theories, something which would clearly give a fatal blow to the association of any physical system to a Turing machine. In general, the computability of a dynamic equation and of initial data does not guarantee the computability of the solution in three distinct cases: when the solutions are not unique, when they are obtained from unbound operators, and when the function representing the solution is neither differentiable nor continuous and is therefore "weak" in the sense of the theory of distributions.[8]

In a word, the software approach to scientific laws does not appear to be sufficiently general to cover the whole range of cases in which laws expressed as differential equations are used in physics. As such, and despite its interest in shedding light on *some* important aspects of scientific laws (their being an economic compression of observational information), it cannot be regarded as an acceptable solution to Wigner's problem and to the problem of explaining in which way laws refer to the world.

6. Measurement as a key to WQ and to understanding nature of laws

Before advancing my own proposal to the problem, let me warn the reader that its appropriate development and defense is the topic of another essay.[9] Here, I can only outline the guidelines of a view that regards measurement not just as the main business of science, but as the key to give a convincing answer to WQ and to the problem of whether and how mathematical laws represent the world.

Let us start by advancing the plausible claim that a *quantitative* treatment of phenomena (or what Suppes calls "data modeling") is a necessary condition of both measurement operations and the applicability of

[8] For a thorough treatment, see Pour-El and Richards, 1989.
[9] See the second chapter of my forthcoming *The Software of the Universe. An Introduction to the History and Philosophy of Laws of Nature*, Ashgate.

mathematics. We should now ask what kind of relationship there might be between the *qualitative* fabric of phenomena – what appears to our senses – and their *quantitative* treatment. Here are some points that I will motivate only schematically, since I regard them as very plausible.

1 Laws *relate* properties P of entities or natural systems. In the examples above, the gravitational force between two bodies is related to their masses and distance, and the pressure and volume of an ideal gas are related to its temperature. *Briefly put, laws are relations between properties P of a system S.*

2 Such properties can become quantitative – scalar or vectorial as they may be, like "having a mass of a certain magnitude" or "having a certain velocity" – only after having introduced a metric (a scale) and having performed some measurements.

3 Since the attribution of particular (real or rational) numbers to properties of events is scale-dependent and conventional, *it is only relations among physical entities that are preserved by their mathematical models.* If I say that "today's temperature is twice as high as yesterday's", my statement is not objective, or rather, has no definite truth value, unless I specify to which measuring system I am referring it (Celsius, Fahrenheit or Kelvin). In any scale, however, what is preserved of the above statement is clearly that today's temperature is *hotter than* yesterday.

4 By recalling Carnap's good old distinction among classificatory, comparative and quantitative concepts,[10] we can regard the distinction between qualitative and quantitative language as a conventional distinction to which nothing in the real world corresponds. This position is tantamount to claiming that the mathematization of the world preserves only some *relevant relations* about the represented natural systems.

5 The key notion to getting close to an understanding of WQ is that of *a partial isomorphic relation between (parts of) a mathematical model and the represented phenomena*, something which entails the existence of a *structural resemblance* between them. Isomorphisms are in fact relations-preserving bijective maps between models and represented phenomena.[11]

[10] See Carnap, 1950.
[11] For an analogous view, see Bueno et al., 2003.

As an illustration of the previous point, suppose we have transitive, asymmetric relations $<_p$ or $<_l$ holding, respectively, among differently plausible versions of witness reports ($<_p$) , or among objects of different length ($<_l$). If we write '$W_1 <_p W_2$' we mean that the report of the witness W_1 is less plausible than the report of the witness W_2, while '$O_1 <_l O_2$' means that O_1 is shorter than O_2. By introducing comparative concepts we are able to refine, order and compare our *intuitive, primitive classification* of, respectively, stories as being "plausible" and "implausible" and objects as being or "short" or "long". We also need equivalence relations "$=_p$" "$=_l$", which partition all our stories and all our objects into disjoint classes, whose members having "same plausibility" and "same length" respectively, whatever our methods to establish such judgements.

The representability in numerical terms of the order that we introduced among versions of witnesses or length of objects by $<_p$ or $<_l$ may be obtained by requiring that real-valued functions P and L satisfy the following conditions:

$$\text{if } a =_p b, \text{ then } P(a) = P(b) \text{ and if } a =_l b, \text{ then } L(a) = L(b)$$

$$\text{if } a <_p b, \text{ then } P(a) < P(b) \text{ and if } a <_l b, \text{ then } L(a) < L(b)$$

$$P(a +_p b) = P(a) + P(b) \text{ and } L(a +_l b) = L(a) + L(b)$$

These conditions create a structural resemblance between numbers and their relationships on the one hand and entities of the real world and their relationships on the other.

Laws intervene at this very stage in pointing out and selecting relationships holding between properties of the phenomena and representing them *via* some numerical function. Considering the importance of the notion of (partial) isomorphism, we can conclude that it is not so much the nature of individuals that matters in such a structuralist reconstruction of the representational capacity of a scientific law, but the fact that some of the relevant relations between individuals are kept by the mathematical model.

In a word, laws are mathematical because we can know only the relational, dispositional structure of events and individual entities, and mathematics is the science of structures and forms. This thesis about the way mathematically formulated laws represent reality is also important to understand the nature of scientific change. As John Worrall has forcefully claimed (Worrall, 1989), what persists unchanged through scientific change is exactly such structures: Fresnel and Maxwell postulated an ether which has been "somehow" abandoned, but their equations are still part of the curriculum of every physics'student.

7. Conclusion

Here are two quotations which will offer me the opportunity for some conclusive comments on the questions that have occupied us till now: "... while structural relations are real in the sense that they are testable, the concept of unobservable entities that are involved in the structural relations always has some conventional element, and the reality of the entities is constituted by, or derived from, more and more relations in which they are involved." (Cao, 1997, p. 5)

In this interesting passage, the historian Cao specifies *the epistemological conditions* enabling us to arrive at a justified judgment about the reality of a theoretical entity ("more and more relations in which they are involved"). The main point of having laws is to specify such conditions in more and more precise way, but it is important to remark that by accepting Cao' thesis there is no need to deny the reality of the entities: relations (which are the object of laws) cannot be born without relata (the entities). In this sense, we should not confuse the mind-independent existence of the theoretical entities themselves with the network of relations, specified by theoretical and phenomenological laws, enabling us to discover them and to attribute them measurable properties.

The second quotation refers to a structural realist understanding of the way laws represent the world that is also an implicit answer to WQ: "To borrow from the ancient philosophical tradition, what I believe the history of science has shown is that on a certain very deep question, Aristotle was wrong and Plato – at least on one reading, the one I prefer – right: namely, our science comes closest to comprehending the real, not in its account of "substances" and their kinds, but in its accounts of the "Forms" which phenomena "imitate" (for "Forms" read "theoretical structure", for "imitate" "are represented by")" (Stein, 1989, p. 52)

Laws are mathematical because the relational structure of many natural system is partially isomorphic to the mathematical structure used to represent them, but the existence of such isomorphisms can only be explained by pointing to the "empirical origin"[12] of some fundamental mathematical concepts, from the geometrical forms that we extract from our perceptions to the experience of temporal succession, which is at the basis of arithmetical counting. The importance of understanding the origin of mathematical concepts with the help of the resources of cogni-

[12]By referring to "empirical origin" of the fundamental mathematical concepts I do not mean to exclude the possibility that such origin be, to use a turn of phrase dear to Spencer, "a posteriori for the specie but a priori for the individual".

tive science may open new paths to the philosophy of mathematics and of natural science in general (see Longo, 2002).

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