Relativistic Becoming and the Conventionality of Simultaneity*

§ 1 Introduction

In the revival of interest in the problem of the conventionality of simultaneity in the special theory of relativity (STR), Malament's theorem [1977] on the unique definability of the so-called "standard" ($\varepsilon = ½$) simultaneity relation in terms of the relation of causal connectibility has been recently subject to severe scrutiny, in particular for what concerns the uniqueness of the simultaneity relation.

In this respect, Sarkar and Stachel [1999] have recently called attention to the fact that Malament's unique definability result might be "sensitive" to small changes in the initial assumptions of the theorem, a point already noted by Spirtes [1981, ch. 6, Sec. F] and Norton [1992, p. 285]. Such an attack to the uniqueness of Einstein's standard simultaneity has stimulated two replies [Rynasiewicz 2001, Giulini's 2001], the second of which appropriately *weakened* the assumptions needed for uniqueness proof, thereby making Malament's result proportionally *stronger*.

While, as I take it, the uniqueness "end" of the debate on the conventionality of standard simultaneity in STR has now undergone a decisive progress, the situation is much more unfortunate with respect to the other "end", involving *the question of understanding what it really takes for a simultaneity relation to be conventional*. The fact that such a question has not yet received a sufficiently clear and non-controversial answer certainly depends on «a

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certain ambiguity in the possible meaning and range of the word ‘convention’» [Giulini 2001, p. 652-3] and Rynasiewicz has stressed the importance of distinguishing «half a dozen distinct forms of conventionalism» [2001, pp.356-7]. However, the situation is not so desperate as it seems, at least if our main purpose lies in understanding the *relevance* of the above mentioned uniqueness results for the question of the conventionality of standard simultaneity.

With this problem in mind, the *first* of the two main points of this note is to point out that the unique definability in terms of some invariant relation *per se* does *not* express a sufficiently strong sense of “non-conventionally simultaneous”: once we understand the notion of “non-conventionally simultaneous” in the right way, the unique definability condition will have to be regarded *at best as necessary* for such a notion, but certainly as *not sufficient*. And yet the *philosophical* import of Malament’s theorem crucially depends on the assumption that a temporal relation be regarded as non-conventional *if* and only if it is uniquely definable in terms of the relation of causal connectibility [1977, p. 293].[1]

I urge that the key to making a significant progress in such an important task of conceptual clarification lies in an unexpected but deep connection between the definability of a becoming relation in Minkowski spacetime and the conventionality of $\varepsilon = ½$, standard simultaneity. Interestingly, the three papers that for a long time provided the only rigorous results in the philosophical literature on the conventionality of simultaneity on the one hand [Malament 1977], and the objectivity of relativistic becoming on the other [Stein 1991 and Clifton and Hogarth 1995], can be shown to be strongly connected by the following conditional claim, which is the *second* and *main* result of this note:

COND: *If there is special relativistic becoming à la Stein, Clifton and Hogarth, the relation of standard simultaneity, defined with respect to an inertial worldline O and any two spacelike-related events, is conventional*.

In interpreting COND, it is extremely important to clearly separate the controversial issue of the *conventionality* of simultaneity from the uncontroversial *relativity* of simultaneity. The former issue amounts to the question whether there is a privileged way of choosing some relation of simultaneity relatively to an arbitrarily selected but already fixed inertial worldline *O*. The latter issue amounts to the universally acknowledged thesis that there cannot be a fact of the matter about which spacelike-related events occur at the same time previously to the choice of a particular inertial worldline. Consequently, COND must take due notice both of the above distinction and of the uncontroversial, non-absolute nature of simultaneity in STR.

Given this plan, in the next section (§2) I will argue in favor of the claim that two spacelike-related events can be *factually* (non-conventionally) simultaneous relative to an inertial worldline *O only if, relative to O, they have become*. For the non-conventionality of simultaneity, the main problem with the italicized, necessary condition is that, as we will see, it turns out to be *incompatible* with special relativistic becoming *à la* Stein-Clifton-Hogarth (henceforth to be abbreviated as 'SCH').

In (§3) I will review the assumptions of SCH's theorems in order to defend them against the superficial accusation that rather than making room for Becoming, they presuppose an eviscerated form of becoming. Furthermore, in order to tackle the objection that the *metaphysical* notion of becoming is as obscure as irrelevant for our understanding of special relativity as a *physical* theory, I will show the importance of a clear analysis of becoming for the foundations and the philosophy of physics (§4-5)

Even from this outline it should be clear that in this paper I am *not* advocating the existence of relativistic becoming, but only exploring its conceptual consequences *vis à vis* the question of the conventionality of simultaneity.

§2The Conceptual Link between Becoming and Simultaneity

In this section I shall argue for the following claim:

C_1 The *O*-simultaneity between any two spacelike-related events is non-conventional *only if* the two events have become relative to *O*.[2]

As a matter of notation, in what follows "*O*" stands for an inertial worldline, "*O*-simultaneous" means "*simultaneous with respect to O*", and "*O*-time" refers to the coordinate time as it is determined by Einstein's standard method of clock-synchronization.

In order to prove C_1, it is sufficient to argue in favor of the following two claims, which unearth the so-far neglected but fundamental link between simultaneity and relativistic becoming:

(1) Einstein's standard relation of *O*-simultaneity between any two spacelike-related events is *non-conventional* (if and) only if such events *occur* at the same instant of *O*-time *as a matter of fact*.[3]

(2) Any two spacelike-related events occur at the same instant of *O*-time as a matter of fact *only if*, relative to *O*, such events have become.

Obviously, (1) $\wedge$ (2) imply the wished-for conclusion C_1.

Prima facie at least, the second premise looks more controversial than the first, which essentially tells us as how we should understand the expression "non-conventionally *O*-simultaneous". But let us examine them in turn.

I take it that when one claims that the standard relation of *O*-simultaneity, in STR, is non-conventional, one means, and should simply mean, that there is *a physical fact of the matter* forcing or at least suggesting our choice of a particular *O*-simultaneity relation. If we agree with Janis that in our context *"conventional" is essentially opposed to "factual"* [1983, p. 101] – and I urge that we should agree with him, also considering Reichenbach's original

interest in distinguishing what in a physical theory is synthetic/factual from what is analytic/conventional – the existence of such a fact about *O*-simultaneous *events* is equivalent in general to the existence of *a mind-independent truth maker*, in our case a *relational fact* or *state of affairs*, linking the two simultaneous events with *O*, and making the particular simultaneity claim true.

Following the suggestion provided by the meaning of "simultaneous" in ordinary language, the relational fact in question would seem to be identifiable with the mind-independent, real *occurrence* of any two spacelike-related events at the same instant of *O*-time as determined by Einstein's criterion, which is (1) above. However, besides the appeal to ordinary language – which in the foundations of a physical theory can be put into question – do we have any additional reasons to cash such a relational fact as we did in (1)?

As is well known, in Malament's theorem such a relational fact is cashed in terms of the relation of causal connectibility and *O*, is unique, and is given by the relation of *orthogonality with respect to O*. Why not accepting Malament's relation as a fact that is sufficient for the non-conventionality of simultaneity, or why not regarding other methods (slow-clock transport) as sufficient? Rather than attacking Malament's definability result or other's, I must stress that here I am only interested in clarifying the *necessary condition* under which we could claim *that there exists some physical fact making a judgment of O-simultaneity true*, and in showing that *such a condition inevitably calls into play temporal becoming*, which is our premise (2).

My claim to this effect is that *any* sort of 3-place relation that is a plausible candidate for being *non-conventionally* instantiated by pairs of spacelike-related events and an inertial worldline *O* – orthogonality with respect to *O* is one of them, a geometrical relation with a physical interpretation, the isotropy of *c* – presupposes a minimal but necessary condition, *namely that, relative to O, the two related events (the relata) have become*. The existence of a

relational state of affairs *in general* presupposes the existence of both its *relata*, or, in short, their co-existence: otherwise, how could we claim that there is a *relational fact* linking the two events relative to *O*? This simple but inescapable remark should also make clear that I am *not* proposing to decide what a physical fact is *via* purely *a priori*, philosophical speculation.

It then follows that for any two spacelike-related events *p* and *q*, and any inertial worldline *O*, *p can* bear *any* simultaneity relation (say, "orthogonality", with its *physical* interpretation) to an event *q* relatively to *O only if p* and *q co-exist or become at the same O-time.* Since it is highly plausible to assume that *the being or the existing of an event is exhausted by its occurring*, the coexistence of *p* and *q* at a particular *O*-time is trivially equivalent to their *co-occurring at that time*, which is (1) above. The condition expressed in (1) is "transcendental" with respect to all other analyses and affects them all, in the sense that any particular candidate for a non-conventional simultaneity relation presupposes it.

Before discussing some conceivable objections to this claim, let me move on to (2) and conclude the argument for C_1. In order to prove the second premise, I must enter the simple but not-sufficiently explored connection between *existence*, *occurring and becoming*. The main reason why debates on the nature of becoming have often been so inconclusive depends, I take it, on our very poor grasp of the intuitive notion of becoming. I propose to define it in the following anti-metaphysical ("deflationary") way:

(DEF) For any event *e* of Minkowski spacetime, *e* "becomes" or comes into existence absolutely just in case *e occurs* or *happens.*

Since the protagonists of temporal becoming must be *events* (or spatio-temporally located state of affairs), and the *coming into being* of events, their existence, lies completely in their *occurring*, for an event to come into being absolutely (or simply "to become") is for it simply to occur.

While (DEF) specifies the meaning of becoming, it is not sufficient to express the simple but essential fact that, *if* becoming has to be more than a mere psychological illusion, along the worldlines that can be traversed by some possible observer *O* (assume for simplicity that *O* is inertial), at least the events that she *lived through* are to be regarded as being *physically different* from those that are yet to happen. Such a physical distinction, however, necessarily presupposes a *relativization* of what exists, and therefore of becoming, to either spacetime points and/or inertial worldlines *O*, in such a way that *becoming must inevitably be regarded as a relation*.

Since a full justification of the claim contained in the last paragraph will be the object of the next section of the paper, for the time being I will assume (i) that becoming involves events – what else could it involve if one is to evaluate whether it is compatible with Minkowski spacetime? – (ii) that, in accord with DEF, it means *occurring*, and (iii) that it is to be regarded as a two or three-place *relation*. For a defense of (2), these remarks suffice.

Recall in fact that (1) claims that the non-conventionality of Einstein's simultaneity relation between any two spacelike-related events entails their *co-occurrence* at a certain instant of *O*-time (their co-existence at that time). Since becoming is to be regarded as a relation, and since – in virtue of DEF – the co-occurrence of spacelike-related events at the same instant of *O*-time is equivalent to their simultaneous "coming into being" relative to *O*, the co-occurrence of any two spacelike related events *p* and *q* relatively to some *O*-time entails that, as of that *O*-time, *p* has become as of *q* and *q* has become as of *p*, which is (2).

Now, putting (1) and (2) together, we get the wished-for conclusion C_1: the non-conventionality of Einstein's simultaneity between any two spacelike-related events and *O* entails a becoming condition between the events and *O*. In order to use it for our purpose, we can put C_1 in the contrapositive, logically equivalent form:

(C) If any two spacelike-related events *p* and *q* have *not* become relatively to an inertial worldline *O*, the events are conventionally *O*-simultaneous.

It obviously follows that *if* we could prove that in STR *no two* spacelike related events can be regarded as having become from each other's perspective relative to any inertial worldline *O*, and if it could be shown that special relativistic becoming as SCH defined it exists, we would have to conclude by *modus ponens* that the relation of simultaneity in STR is purely *conventional or non-factual*. Such a conventionality would descend from the fact that in special relativistic universes with becoming, any event *q* that is spacelike-related to any event *p* simply has not become with respect to *p* relative to any inertial observer *O* going through *p*, and therefore does not exist as of *p*. *Consequently, there cannot exist any physical fact that, relative to O, serves as truth-bearer for the simultaneity claim established with some synchronization method.*

A final objection needs to be considered to the whole project of this paper: in view of Quine's criticism of the distinction between matters of convention and matters of fact in science, one should resist any Reichenbach-type attempt to separate facts from conventions. While I admit that in general the distinction between analytic (non-revisable) and synthetic (revisable) truths is definitely less rigid than the neopositivists thought it to be, in the case of the relation of simultaneity the difference in question seems evident: is there a fact suggesting or forcing us to choose one among the infinite numbers that epsilon might take in the well-know relation linking the reflection time t_r with the emission time t_i and the final return *time* t_f?

$$t_r = \varepsilon\left(t_f - t_i\right)$$

§2.1 Four objections to (1) and (2)

In judging the appropriateness of the definition of non-conventionally *O*-simultaneity given in (1), one should go back to the original point of Einstein's 1905 stipulation ("*Festsetzung*"), which was to provide observers sharing the same inertial motion an operational criterion to assign the *same temporal coordinate* to all events in the relative three-space. Bearing this purpose in mind, one might want to object that (1) relies on too *metaphysically* loaded an understanding of *O*-simultaneity.

One might then want to distinguish between *two different senses* of "*O*-simultaneous", a weaker one given by "sharing the same time-coordinate relative to an inertial worldline *O*" – call this "*O*-simultaneous$_1$" – and the more robust sense I am proposing in (1), call it "*O*-simultaneous$_2$", given by "*occurring or happening* at the same instant of *O*-time". The basis of this distinction is that, *prima facie*, *O*-simultaneous$_2$ obviously implies *O*-simultaneous$_1$, while the converse implication might *not* hold, especially if we assume with Eddington [1920, p. 51] and Weyl [1949, p. 116] that in STR "events *do not happen* but *simply are*".

Granting for the sake of the argument that such a distinction between these two senses of "*O*-simultaneous" is plausible, I should discuss four related objections that could be leveled against (1) and therefore against C:

(i) Einstein's original purpose of synchronizing two clocks at rest in the same inertial system was given by the need of introducing a well-defined notion of *velocity* relatively to that system. With the notion of velocity in hand, one can refer to other inertial frames and Lorentz-transform among them: from this viewpoint, the concept of relative inertial velocity would matter more than simultaneity$_1$. Calling the events corresponding to the

same readings of two synchronized clocks "simultaneous$_1$" might then correspond to what Grünbaum calls a "trivial semantic convention", a mere choice of words.

(ii) the sense of simultaneous that is relevant for doing physics, or sufficient for the purpose of describing the physical world, is "sharing the same instant of *O*-coordinate time" (simultaneity$_1$) given some method of synchronization; consequently, the true question about the conventionality of simultaneity in STR is whether that method picks up, or is suggested by, a physical fact making the simultaneity claim true.

(iii) The whole debate on the conventionality of the *O-simultaneity* has so far been referring to simultaneity$_1$ and not to simultaneity$_2$.

(iv) Simultaneous$_2$ is so *strong* a sense of "simultaneous" that it only makes sense in a *Newtonian* setting, i.e., it presupposes the *absoluteness of simultaneity*, or its being an invariant notion, a presupposition which in STR obviously *cannot* be granted.

If correct, these objections might force us to regard the introduction of the notion of becoming in the discussion about the conventionality of simultaneity as inappropriate. At best, it would turn our discussion into a purely terminological debate, or a wordy warfare. Let us discuss these four points in turn.

(i) If calling the events that share the same *O*-time (on the basis of Einstein's method) "simultaneous" were really a matter of arbitrarily choosing some words rather than others, then there would be no point in trying to argue that Einstein's simultaneity relation is *non*-conventional: we would have solved the question *by fiat* in favor of the conventionality of simultaneity. On the contrary, trying to establish whether two events sharing the same *O*-time are simultaneous in some non-trivial, conventional sense or not is *essential* to the foundations of STR. Historically, it is highly plausible to assume that the greater obstacle Einstein had to

overcome in order to formulate STR lied in accepting the relativity of simultaneity, which was indispensable to postulate the invariant character of *c*. One could not even claim, as it is usually done, that the relativity of simultaneity is one of the major innovations of STR with respect to Newtonian mechanics if the use of "simultaneous" in STR emerged from a mere choice of words and did not bear some continuity with "simultaneous with" as it was used in theories in which such a relation *was absolute*.

(ii). Even if, for the sake of the argument, we were to grant that the distinction between the two senses of simultaneous is worth-making, (ii) generates a dilemma. Either "sharing the same instant of *O*-time" is too weak a notion to bear the claim that Einstein's standard relation of *O*-simultaneity is *non*-conventional, or else, if it is strong enough to bear that claim, the true question becomes whether Einstein's (or any other) method of assigning the same time-coordinate relative to *O* picks up a fact forcing or suggesting us to choose that particular method.

If we take the first horn, our explication or analysis of the sense in which *O*-simultaneity is *factual* as opposed to *conventional* has simply failed, and the dispute about the non-conventionality of simultaneity becomes meaningless; as a consequence, if we think that the dispute is meaningful, we need to adopt the more robust notion of simultaneity$_2$ and therefore accept (1), (2) and (C). If we take the second horn, we are in any case falling back on the analysis proposed in the previous section: if there is a physical, relational fact suggesting the choice of one method of synchronizing clocks at the distance over others, *we will have to presuppose that, relative to O, the related events exist*. Otherwise, on what ground could are we to assume that such a *relational fact exists*? It then follows that also the attribution of the same *O*-temporal coordinate to pairs of spacelike-related events (i.e., simultaneity$_1$) *presupposes as a necessary condition* that, relative to *O*, the two events have become (are real, or definite) as of each other's perspective.

Given that any reasonable understanding of a non-conventional *O*-simultaneity takes us back to (1), the objection (iii) loses its bite too. Even assuming that the discussion of the conventionality of simultaneity has so far focused on simultaneity$_1$, this has happened simply because so far we have been taking for granted that relative to *O*, all the events that are spacelike-related to any event *e* have become. However, this means that we have not yet made explicit the essential conceptual connection between becoming and simultaneity, and the extent to which such a connection clarifies the issue of what it takes for the simultaneity relation to be conventional.

As to the objection (iv), note that if the *relativity* of simultaneity entailed its *conventionality* – or, in contrapositive form, if a non-conventional simultaneity relation entailed its frame-independence – given the uncontroversial acceptance of the relativity of simultaneity, I would be reaching the conclusion I am after (the conventionality of simultaneity) by following a different path. If in STR we required that any relation can be factual or non-conventional *only if* it is invariant, then simultaneity would be conventional by definition, since in STR it is a relative notion. For the time being, however, I think that we should stick to the oft-accepted view that "being relative to an inertial worldline *O*" *per se* need *not* imply that there cannot be mind-independent *facts* relative to *O* suggesting the choice of a particular simultaneity relation (Einstein's, in our case). So simultaneity$_2$ – that specifies the condition under which it is possible to claim that there is a fact of the matter about which events are *O*-simultaneous – need *not* imply the absoluteness of simultaneity, since not all facts in special relativity are invariant facts: time-dilations or length contractions are perfectly measurable and in this sense real "facts", despite their "relative nature". The reader that is still unconvinced by these examples may regard what follows as a new argument in favor of the claim that the relativity of simultaneity entails its conventionality.

§3 The Meaning of Becoming and SCH theorems

Given the disagreement surrounding the notion of becoming, however, a critic may object that we have made very little progress in our problem. If we want to understand the unclear notion of "conventionally simultaneous" with the help of becoming, we might simply be replacing the obscure with the mysterious. Furthermore, even if – as we are about to see – Stein's and Clifton and Hogarth's theorems, together with (C), imply that *if* there is becoming as they defined it, the relation of standard simultaneity is purely conventional, *how can we establish on purely physical grounds whether in fact there is becoming in STR?* In other words, what is the relevance of becoming for our understanding of Minkowski spacetime? An answer to these questions presupposes a defense of the approach of becoming originally advanced by SCH.

Since DEF equated the claim that the "becoming" of *an* event is objective with the "minimalistic" claim that such an event *occurs* independently of our minds, the becoming of *a set of temporally separated events* in STR must consist in the fact that *such events come into existence – i.e. occur – successively*. Except for the reference to a "now", note that this approach closely follows Gödel's: «The existence of an objective lapse of time means or at least is equivalent to the fact, that reality consists of an infinity of layers of "now" which come into existence successively» [1949, p. 202]. In order to show why the special relativistic "now" is reduced to a point-event, we must connect this analysis of becoming with Stein's relation of "being definite with respect to a point" and with Clifton's and Hogarth's "worldline-dependent becoming", which is clearly more relevant for our purpose of meshing the relation of becoming with *O*-simultaneity.

In the original formulation due to Stein, a becoming relation should be regarded as a binary *relation* holding between events or points in a worldline-independent fashion. So for Stein *aBb* simply means that "event *a* has become as of event *b*". In Clifton and Hogarth's extension of Stein's approach, one considers a worldline *O*, not necessarily followed by actual observers (and not necessarily inertial, though for our purposes we can restrict our attention to inertial trajectories), and stipulates that aB_Ob mean "event *a* has become for an observer following the worldline *O* and coinciding with event $b \in O$" [Clifton and Hogarth 1995, p. 363].

In order to show that Minkowski spacetime is compatible with becoming, we must require that, as of any event *a* in the spacetime, there must be some event that has not yet become as of *a*. This is exactly the first of Stein's hypotheses of his compatibility theorem: «For any point *a* [of Minkowski spacetime], there are points whose state is still unsettled as of *a*» [1991, p.148]. For consistency with the terminology I have chosen so far, Stein's "settled" and "unsettled", "definite" and "indefinite", will be defined in terms of becoming ("has become", "has not become" respectively). Likewise, I will assume that we can identify points with events, since nothing crucial hinges upon this distinction, so that I will translate Stein's above premise with the assumption that for any event *a* of Minkowski spacetime, there are events that have not become as of *a*. Which other premises is it reasonable to assume?

Given the analysis above, we are now justified in following SCH and regarding becoming as a *binary relation* holding either between pairs of spacetime events (Stein's absolute becoming) or spacetime events and points on any worldline *O* of spacetime (Clifton and Hogarth's worldline-dependent becoming). Clearly, it then follows that any event *a* has trivially become as of itself (*a* "has occurred" as of *a*), and that if the event at point *b* has become as of *a*, then whatever has become as of *b* must (*a fortiori*) have become as of *a* (see

Stein [1991, p. 148]).

Thus the becoming relation *B* has to be regarded as *reflexive* and *transitive* in virtue of its meaning, and in virtue of the fact that in SCH's *invariant* approach to becoming, the *objectivity* of the relation *B* necessarily requires the *intersubjective agreement* as to which events have become for *any* two inertial observers *O* and *O'* being in relative motion. In general, transitivity means that if aB_Ob and $bB_{O'}c$ then $aB_{O'}c$ for any inertial worldline *O* and *O'* and for any event *a b* and *c*. If *b* = *c*, we have the interesting case in which the two observers traveling on *O* and *O'* "meet" at a point *b*. Clearly, if becoming has to be objective, they must agree as to which event has become as of *b*, but if *R* were non-transitive, we would have the implausible consequence that such observers, approximately sharing the "same" here-now (locally perceiving the same events), would disagree as to which events have become at a distance! This is the main reason to reject a relativization of a becoming relation to the 3-space individuated by an inertial frame, against the opinion of many scholars [see for one Putnam 1967].

In short, *if* we want becoming in STR – and do not want to tinker with the theory by adding to the structure of Minkowski spacetime an empirically otiose but privileged reference frame [as in Rakic 1997] – either we give up the extendedness of the present (with SCH), or we give up the transitivity of what has become for different inertial observers. Which option should we choose in cases like these is really a question that amounts to finding the *best reflective equilibrium* between the most fundamental intuitions of the world of our experience on the one hand (the "manifest image" of the world), and the requirements of the "scientific image" of the world on the other [Sellars 1962]. If we encountered difficulties in explaining our experience of the world within a block universe, and we wanted to save becoming, we should consider that it is not impossible to explain why we could end up *having the psychological impression* that the present instant extends across the boundary of space: below

30/40 millisecond, we cannot tell *two* temporally successive light-signals apart, and in that interval light covers from 9000 to 12.000 km![4] On the other hand, it is *not* plausible to abandon the transitivity of "has become for", since then the intersubjective validity of becoming would be completely lost. On the hypothesis of becoming, we must keep the transitivity of becoming and give up the spatial extendedness of the present.

Two further, crucial assumptions are needed: the first is that Minkowski spacetime be endowed with a coherent distinction between a future pointing and a past pointing direction of time, which is tantamount to impose to the spacetime a distinguished temporal orientation[5], necessary for definability and the consistency of becoming. The second assumption needed to prove the theorem is that any point *a*, an event at can be influenced only by events in the causal past of *a*, namely only by events that, being in the past light-cone of *a*, intuitively must be regarded as having already become as of *a*: (see Stein [1991, p.149]). Analogously, for any *O* and $b \in O$, if $a \in O$ is in the timelike past of *b*, then aB_Ob for any *a* [Clifton and Hogarth, ibid.]. After all, had events in the past light cone of *b* (in the timelike past of *b*) not become as of *a*, how could they possibly be influencing or causing *a*? If we did not assume that an explosion of a star has occurred in the causal past of the event coincident with our *present* observation in the telescope, how could we explain what we see?

Here, the question of becoming overlaps non-trivially with the controversial and complex issue of *the arrow of time*, and in fact the above asymmetrical assumptions might be criticized in virtue of the fact that they arbitrarily assumes that the metrical relations are *time-asymmetric*: it is because we take for granted that *only* events in the causal (timelike) past of *a* can *influence a* that we are justified in assuming that all such events have become with respect to *a*. Agreed: in current physics, characterized by time-reversal invariant physical laws (with the exception of CP violations in weak interactions), a time-symmetric relation of causal (or

timelike) connectibility cannot be ruled out *a priori*. However, I take it that until we have *clear, non-controversial* cases of backward causation, we are entitled to assume that, at least *de facto*, causes always precede their effects, so that we have no traces of the future in the present, but only traces of the past: for our purposes, this will do.[6]

In conclusion, we can finally formulate Stein's theorem and derive immediately the corollary which interests us: «If *R* is a reflexive, transitive relation on a Minkowski space (of any number of dimensions - of course at least two), invariant under automorphisms that preserve the time-orientation, and if *Rab* [*b* is definite as of *a*, my addition] holds for some pair of points (*a*, *b*) such that *ab* is past-pointing (timelike or null) nonzero vector, then for any pair of points (*x*, *y*), *Rxy* holds if and only if *xy* is a past-pointing vector» [1991, p. 149].[7]

Noting the *absolute*, frame-independent nature of the becoming relation defined by Stein's theorem, it could be objected that Malament's and Stein's result cannot "mesh" as proposed in COND, since the former theorem makes crucial reference to a worldline *O*, while the latter doesn't. In order to counter this objection by using Clifton's and Hogarth's extension of Stein's theorem to all worldline-dependent becoming relations, consider the set *CC* of points in the causal past of *a* that are *chronologically connectible* with *a* (that is, remove the light-like separated events from the set of events that can cause *a*).[8] Such a set *CC* can be generated or covered by all *inertial* worldlines *O* passing through *a*; if *all* observers traversing such *O*'s that pass through *a* agree – as they should given the transitivity of worldline-dependent becoming B_O assumed above – that *b* has become with respect to *a* relative to their particular *O* going through *a*, then we can assume that *b* has become as of *a* *absolutely* (independently of a relativization to a particular worldline *O*). Conversely, "*aBb* can reasonably be taken to imply that aB_Ob for every *O* going through *b*" [Clifton and Hogarth 1995, p. 362].

Given this connection between absolute and worldline-dependent becoming, Clifton and

Hogarth prove that if the collection of worldline-dependent becoming relations associated with all possible observers in a time oriented Minkowski spacetime satisfy the above premises, then "every becoming relation in the collection is either the relation of past chronological connectibility or Stein's relation of causal connectibility or the universal relation" [ibid., p. 372-3].

Clearly, *if* we want becoming, we must exclude the universal relation and we are left with the other two time-oriented metrical relations. Likewise, *the non-universality of B is a premise of Stein's theorem* that is crucially used in the proof. To the extent that we assume, correctly, that the existence of becoming in STR is equivalent to the claim that for any event *a* there are events that as of *a* have not become, one could accuse Stein's theorem of circularity. However, this charge would be completely groundless: *rather than in proving the existence of a becoming relation in Minkowski spacetime, SCH's interest lies in showing its compatibility with the structure of such a spacetime*. In a word, SCH's result (and as a consequence also COND) have a *conditional* nature: *if* there is becoming (i.e., *B* is non-trivial and non-universal and satisfies the other plausible assumptions listed above), *then* for any event *e* in Minkowski spacetime, the set of events that has become relatively to *e* coincides with the causal past or the timelike past of *e*.

Given that the relations of becoming is *defined* in terms of the relation of past causal connectibility or past chronological connectibility, it follows immediately that two events have not become as of each other just in case they are not causally connectible or just in case neither is in the chronological past of the other. Consider Stein's absolute becoming: since *aBb* if and only if *b* possibly causes *a*, and *bBa* if and only if *a* possibly causes *b*, it follows that two events are *not* causally (timelike) connectible (*a* cannot possibly cause *b* and conversely) if and only if they have not become as of each other. Since a similar argument

goes through for worldline-dependent becoming, the corollary we were after is proved:

COR: If there exists worldline-dependent becoming in STR as defined by SCH, *any* two spacelike-related, causally non-connectible events *p* and *q* have not become relatively to any inertial worldline *O* going through either *p* or *q*.

In fact, if we consider any inertial worldline *O* and a point *a on* it, the set of events that have become relatively to *O* as of *a* will include any point on the *chronological past* of all such worldlines *O*, but no point that is spacelike related to *a*. It is possible to show that if only one spacelike related point had become relative to some point *p*, the only relation that would be left would be the universal relation [Clifton and Hogarth, 1995, ibid.]. Clearly, the asymmetry of the becoming relations is their central feature: being reflexive and transitive, if they were also symmetric, they would become equivalence relations, and it is well known that the only becoming relations that are equivalence relations in Minkowski spacetime are either the trivial relation (according to which each point has become as of itself) or the universal relation!

We can now go back to the issue of the conventionality of simultaneity with respect to an arbitrarily selected inertial worldline *O*. Considering that, on the basis of (C) above, the co-existence of any two spacelike-related events [their having become as of each other's perspective] is a necessary condition for their being non-conventionally simultaneous, we have also proved the conditional we were after:

$$[\text{COR} \wedge \text{C}] \rightarrow \text{COND}$$

COND: *If there is becoming in Minkowski spacetime as SCH defined it, then standard simultaneity is conventional*.

In fact, the existence of becoming implies, in virtue of COR, that relative to any inertial worldline *O*, any two spacelike-related events have not become as of each other's perspective, and (C) tells us that if this is the case, such events cannot be factually simultaneous, for the

reasons given in the previous section. It follows that if there is relativistic becoming, standard simultaneity is conventional.

§4 Some objections to the significance of COND for the conventionality of simultaneity

Even by recalling the important qualification that for the overall purpose of this paper, no defense of the reality of becoming is needed, I should certainly consider some objections to the conditional result stated above, the first of which involves the question why – *on the hypothesis of becoming* – one should conclude that spacelike-related events picked out by Einstein's method are *conventionally O*-simultaneous, rather than *not simultaneous at all*. Given the corollary to Stein's theorem (COR), wouldn't it be more appropriate to conclude that the relation of simultaneity is never instantiated in a Minkowski spacetime with becoming? My reply to this question is positive and consists in biting the bullet: *relativistic becoming conjoined with the relativity of simultaneity implies that there is no matter of fact about simultaneous events*, so that the relation of simultaneity strictly speaking is never instantiated.

However, for the purpose of doing physics, one certainly needs to assign temporal labels to the region of spacetime that is not casually connectible with respect to a point on *O* coincident with some inertial observer. It is for this very reason that we need to introduce a *convention* (Einstein's *Festsetzung*), that consists in *constructing* (*via* Einstein's method, or some other one) a *fictitious* set of events lying in a 3-space, each of which can be imagined to co-occur with a given event lying on *O*.

A second difficulty that could jeopardize the significance of COND is generated by the fact that even in universes with becoming, we might be able to claim that *with respect to*

events q in the future light-cone of O's here-now p, all events *E* contained in the causal past of *q and* lying in the hypersurface orthogonal to *O* that goes through *p were* non-conventionally simultaneous with *p*, even though, *as of p*, no spacelike-related event has become or is definite relative to it. Being in the past light-cone of the future event *q*, the events *E* have all become as of *q* and one might be tempted to conclude that they are therefore "definite" enough to bear simultaneity relations to one another and to *p*. Therefore, it could be objected, also the events in the past light-cone of *p can* be non-conventionally simultaneous with each other according to Einstein's criterion: the SCH theorems guarantee that *as of p* there must be a fact of the matter about which events *were* simultaneous with a given event *r* in *O's chronological past*. After all, Einstein's radar-method presupposes that in order to judge which event *r* on our past worldline *was* simultaneous with the reflection of the light ray by the distant mirror, we must wait until the reflected light ray intersects again our worldline at the here-now *p*: as of *p* both events are in our causal past and according to SCH both have become as of our here-now.

However, a little reflection shows that also this objection does not deprive COND of its force. Suppose that *v* and *w* are two spacelike-related points in the causal past of *p*, lying in a hypersurface *S* that is orthogonal to the observer *O* whose here-now is *p*. Clearly, any event lying in *S* has become as of *p*, and is therefore definite *as of p*, but according to SCH *v* and *w* are not "definite enough" to bear some physical relation to each other, simply because they are *causally non connectible*. Consequently, given SCH's theorems, events in *S*, though in the timelike past of *p*, *have not become as of each other's perspective* also from the standpoint of *p*, as the becoming relation is transitive but not symmetric.

In sum, if we are going through *p* and ask ourselves whether *there was* any fact of the matter about which events really co-occurred with the event *q* in the chronological past of our worldline *O*, the answer to this question must *NO* because, on the hypothesis of becoming, as

of *q there was no fact of the matter about events that are causally non connectible with q*. We shouldn't forget that any becoming claim should be interpreted in a strictly *relational* manner: consequently, in our setting there is no absolute, perspective-independent notion of "being definite", and what count as "real" is irreducibly relational. Therefore, the claim that "as of *p* there is (tenselessly) a state of affairs concerning the simultaneity of events in the *past* light-cone of *p*" is covertly *epistemic*: rather than "there is a state of affairs" we should more correctly say: "as of *p* there is information that could in principle allow us to make claims about past events".

Going now to a last difficulty, note that the request that events and the spacetime locations at which they occur be *distinct* may jeopardize the relevance of COND for the issue of the conventionality of standard simultaneity in STR. In fact, while the identification of events with spacetime points presupposed above has the consequence that *neither events nor spacetime points* can be the *relata* for a simultaneity relation because they have not reciprocally become, if we distinguished events from their spacetime locations, the situation could be different. The non-existence of spacelike-related *physical events* that have become as of each other's perspective is compatible with the fact that purely geometrical, "unoccupied" spacetime points may play the role of such *relata*. Such an option seems somehow to be committed to a version of (Minkowski) spacetime *substantivalism*, at least to the extent that the doctrine of substantivalism entails that spacetime points exist independently of the events occurring in them. If this possibility were to be taken serious, the claim put forth by COND would rely on some version of *relationalism*, namely that all spatiotemporal relations characterizing Minkowski spacetime are parasitic upon physical relations exemplified by physical events.

However, a little reflection shows that swallowing Minkowski's manifold

substantivalism in order to attack COND is too big a price to pay: if in a theory in which space and time are *not* an inert background on which events occur (the general theory of relativity), the substantivalist is forced to embrace *metrical*-field substantivalism rather than the objectionable manifold substantivalism (see Hoefer 1996, Maudlin 1988, Stachel 1993), it would seem pretty outlandish to endorse a distinction between events and spacetime points in *Minkowski* spacetime in order to admit in one's ontology unoccupied spatiotemporal locations.[9]

Therefore we can conclude that *if* we restrict ourselves to universes in which becoming is real, spacelike-related events cannot be objectively, factually simultaneous. The question of whether it is viable to derive the conventionality of simultaneity by using COND and *modus ponens* now poses itself naturally. In order to answer it, we must first pose the problem of the *relevance of becoming to physics* and then place the issue at stake in a wider philosophical perspective.

§5 The relevance of becoming to physics and the conventionality of simultaneity

Post-factum, it is not unreasonable to claim that Einstein's original reference to a necessary stipulation concerning simultaneity at a distance [1905, pp. 893-4] can be reconsidered and better understood by contemplating, more attentively than it has been done up to now, the consequences of assuming that the *relativistic present is restricted to a point (the "here")*. Assuming the reality of relativistic becoming forces us to embrace a consequence that might be implicitly derivable from the relativity of simultaneity alone:[10] if there is no fact of the matter about which events occur at a distance because *no* such event has become with respect to our here-now, taking steps toward the *relativity of simultaneity* – unquestionably the greatest conceptual innovation of STR – will appear *much easier and much more plausible* than it would otherwise look if we were convinced that the *present* has

an objective, cosmic extension as it was in Newtonian spacetime. The last paragraph by itself seems sufficient to justify the relevance of becoming for physics and its foundations: if I am right about becoming as a necessary condition for a non-conventional simultaneity relation, it is not an exaggeration to claim that without SCH's theorems, we would have not gained a deeper understanding of the *meaning* of simultaneity in STR.

Secondly, the link between becoming and the arrow of causation, implicit in Stein's assumption of an *asymmetrical* relation of causal connectibility, is also a link with important foundational questions concerning the so-called "arrow of time". For instance, can we assume that the relation of causal connectibility is time-asymmetric in virtue of purely *de facto* asymmetrical processes (entropy growth, pervasiveness of retarded versus advanced radiation, expansion of the universe, violation of T-invariance in weak interactions)?

Nevertheless, it must be admitted that any physical evidence in favor of becoming must be *indirect*, since it must come from evidence in favor of a time-asymmetric relation of causal connectibility, in terms of which becoming is defined. If we lived in a universe in which physics would force us to admit that we have traces of the future in the present (because, say, of the existence of close time-like curves together with the time reversal invariance of physical laws), becoming as SCH defined would be impossible or not definable. Analogously, if quantum non-locality forced us to conclude in favor of a *symmetrical* causal dependence of two spacelike-related measurement outcomes in Bell-type correlations, a necessary condition of relativistic becoming would be violated (Dorato 1996, Myrvold 2003). Since we can reject the existence of a becoming by invoking parts of physical theories that force us to assume a symmetrical becoming relation, becoming is not at all irrelevant to physical theorizing.

However, since any physical evidence in favor of becoming has to be indirect, we cannot try to establish *directly* in physical terms whether it is legitimate to assume that, for

any point *e* of Minkowski spacetime, some events have occurred as of *e* and some haven't. Despite this limitation, however, our interest in the definability of becoming in Minkowski spacetime is legitimate to the extent that the problem of establishing the compatibility of the time of physics with the time of our experience is a fundamental problem of contemporary philosophy of physics.

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[1] In fairness to Malament, he is fully aware of the fact that this may not be the only acceptable analysis of "conventional simultaneity" and seems non-committal about its value. He qualifies it as "a version of the causal theory of time", and attributes it to Grünbaum, who has recently disclaimed it [2003].
[2] The precise meaning of "becoming" will be clarified below.
[3] In what follows I will limit my attention to "only if" part, which is sufficient for my purpose. However, (1) is supposed to be a definition of non-conventionally simultaneous.
[4] This explanation is sketched in Stein [1991].
[5] For this notion, see Earman [1974] and Wald [1984], p.189.
[6] In any case, only general relativistic spacetimes seem to be amenable to time travel and here we are focusing on STR. For an extensive and brilliant defense of time-symmetric physics, see Price [1996]. See also Savitt [1995] and Albert [1999].
[7] Automorphisms can be regarded as symmetries of the spacetime *M*, that is, as bijective maps of *M* onto itself that preserve its causal structure.
[8] For a more accurate definition of the distinction between chronological future (past) and causal future (past), see Wald [1984, p. 190].
[9] It could be countered that one needs to distinguish between points and events at points in order to use the existential quantifier and be able to claim that: «For any point *a* [of Minkowski spacetime], *there are* points whose state is still unsettled as of *a*» [1991, p.148]. But if such points are concrete entities, then they are simply events, and if they are not, they are abstract, causally inert entities, which cannot be the relata of a concrete, physical relationship.
[10] Before SCH, this claim was clearly realized already by Robb:«the present instant, properly speaking, does not extend beyond itself.» [Robb 1921, p. 7].