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CORPORATE VALUATIONS AND THE MERTON MODEL

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Abstract. *In recent years both practitioners and academics have realised that traditional discounted cash flow models erroneously consider the option value embedded in firms. Hence equity and debt valuation methodologies based on option theory have recently become quite popular. Such methodologies take inspiration from the Merton (1974) model which was originally introduced to measure the impact of default risk on corporate bonds yields. Thirty years later the Merton model for its simplicity and rigour remains unrivalled and is the basis of some of the most sophisticated credit risk models. In this paper it will be shown how practitioners often improperly adapt the Merton model for aims beyond its original scope.*

I. Merton Model and Corporate Valuations

The Merton model is a credit risk structural model and assumes that if assets are not sufficient to meet the obligations the firm defaults. Its original aim is to determine the appropriate yield for a corporate bond issued by a firm subject to default risk.

The model is continuous and assumes that the only source of uncertainty that influences the firm's ability to pay in every time t is its asset value $V(t)$. Its stochastic evolution is expressed as:

$$dV(t) = (\mu_V V(t) - c_V)dt + \sigma_V V(t)dz \qquad (1)$$

where

μ_V is the expected return rate of $V(t)$ per unit of time

c_V is the payment received or remitted by the firm per unit of time

($c_V < 0$ in the case of new financing , $c_V > 0$ in the case of expenditure[1])

σ_V^2 is the instantaneous variance of the asset return per unit of time

dz is a standard Wiener process

Assets can be financed through debts or equity, therefore:

$$V(t) = D(t) + S(t) \tag{2}$$

where $D(t)$ and $S(t)$ are the value of the debt and the equity at time t. Supposing that $D(0)$ is the cost at the time $t = 0$ of a zero coupon bond with face value B and maturity T and the firm is unable to issue new debts or pay coupons or dividends ($c_V = 0$), the equity value at time T is represented by the payout of a European call option on $V(t)$ with strike price B:

$$S(T) = \max(0, V(T) - B) \tag{3}$$

This value at time 0 can be found using the Black–Scholes (1973) formula as:

$$S(0) = V(0)N(d_1) - Be^{-rT}N(d_2) \tag{4}$$

where

r is the risk free rate

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[1] For example, dividends or coupon payments.

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left[-\frac{1}{2}y^2\right] dy$$

$$d_1 = \frac{\log\left(\frac{V(0)}{B}\right) + \left(r + \frac{1}{2}\sigma_V^2\right)T}{\sigma_V \sqrt{T}}$$

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

The debt value can be found by difference $D(0) = V(0) - S(0)$ and the debt yield i^* by solving the equation $D(0) = B \cdot (1 + i^*)^{-T}$.

It must be stressed that the use of the Merton model is straightforward when the debt of the firm is a single zero coupon bond, but otherwise it does not provide closed formulae for corporate valuations.

In Damodaran (1999) it is shown how to use the Merton model in its simplest form to value debt and equity of corporations with any capital structure. When doing this there are several practical issues that need to be addressed. In particular, in this paper the attention will be focused on the three major ones; two are with regard to inputs of the model, namely the asset value volatility σ_V and the asset value $V(0)$, and the remainder is in reference to the adjustment procedure used to consider the debt as a single zero coupon bond when it is not.

II. The Asset Volatility

The most critical input for options and hence also for real options valuation is the underlying asset volatility. In order to estimate the variance of firm returns practitioners often use the following relation:

$$\sigma_V^2 = \left(\frac{S(t)}{V(t)}\right)^2 \sigma_S^2 + \left(\frac{D(t)}{V(t)}\right)^2 \sigma_D^2 + 2\frac{S(t)D(t)}{V(t)^2}\rho_{SD}\sigma_S\sigma_D \qquad (5)$$

where

σ_S^2 is the instantaneous variance of the equity return per unit of time

σ_D^2 is the instantaneous variance of the debt return per unit of time

ρ_{SD} is the correlation coefficient between equity and debt returns

This equation can be misleading. In fact like any other model, the Merton model should not be used without taking into account its explicit and implicit hypotheses. Considering equation (1) and (2) and the fact that the unique source of uncertainty in the model is the asset value $V(t)$, the implicit dynamics of debt and equity values $D(t)$ and $S(t)$ are:

$$dD(t) = \mu(t, D(t))dt + \sigma_D D(t)dz \qquad (6)$$

$$dS(t) = \mu(t, S(t))dt + \sigma_S S(t)dz \qquad (7)$$

where $\mu(t,D(t))$ and $\mu(t,S(t))$ are the debt and equity value drift terms.

Since from equation (2) it can be inferred that

$$dV(t) = dD(t) + dS(t) \tag{8}$$

it follows that the relation between σ_V and stock and debt volatilities σ_S and σ_D must be:

$$\sigma_V = \sigma_S \frac{S(t)}{V(t)} + \sigma_D \frac{D(t)}{V(t)}. \tag{9}$$

Therefore Merton model's hypotheses implicitly assume that the correlation coefficient ρ_{SD} between the random rates of return of variables S and D is 1. This gives a definitive answer to one of the "estimation" questions posed in corporate finance when dealing with this framework: the correlation coefficient ρ_{SD} is not a variable but a constant which does not need to be estimated.

III. The Asset Value

An actual estimation issue is about the asset value $V(0)$. For this purpose practitioners usually discount expected cash flows at the weighted average cost of capital (WACC) which is an increasing function of the asset volatility.

Therefore if the WACC is used to find the initial asset value $V(0)$, the resulting equity value $S(0)$ is a compound function of the asset volatility σ_V:

$$S(0) = f\left(\sigma_V, V(0)\right) \quad \text{where} \quad V(0) = f\left(\sigma_V\right) \qquad (10)$$

Hence, at any time t, the sensitivity of the equity value $S(t)$ to variations of σ_V can be expressed as:

$$\frac{\partial S(t)}{\partial \sigma_V} + \frac{\partial S(t)}{\partial V(t)} \cdot \frac{\partial V(t)}{\partial \sigma_V} \qquad (11)$$

Equation (11) differs from the *vega*[2] of a financial option since it includes the additional term $\frac{\partial S(t)}{\partial V(t)} \cdot \frac{\partial V(t)}{\partial \sigma_V}$. This term is negative since it is the product between the call's *delta* $\frac{\partial S(t)}{\partial V(t)}$, which is positive, and the sensitivity of the asset

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[2] The option's *vega* is the rate of change of the option value with respect to the underlying asset volatility. In this case the option considered is the equity.

value to variations of the asset volatility $\frac{\partial V(t)}{\partial \sigma_V}$, which is negative. The latter is negative because the higher the asset volatility, the higher the equity risk premium[3] is (hence the WACC) and the lower is the resulting asset value.

Therefore when valuing the equity as a call option it is not appropriate to state that the equity value is always an increasing function of the underlying asset volatility as it is with financial options.

This is also in contrast with some real options principles which however do not seem to be correct from a common sense point of view[4]:

- The higher the uncertainty of the economic result of a firm (or a project) the higher the value of the call option;
- Flexibility in the structure of a firm (or a project) always makes the call option more valuable.

The remarks contained in this section provide a more analytical explanation as to why the above statements are inaccurate.

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[3] The equity risk premium is defined as the difference between the cost of equity and the risk-free rate.

[4] For more details see Mayor (2001).

IV. Synthesis of the Debt

The debt structure of a company is very seldom as simple as a single issue of zero coupon bonds. However in order to use the Black–Scholes formula to find the price of the call option held by the stockholders when the debt structure is more complex, a transformation method to convert such debt structure into an "equivalent" single issue of zero coupon bonds is required. Unfortunately this synthesis process is not acceptable from a probabilistic point of view and leads to a great discrepancy from the real world.

In fact considering the transformation methodology proposed in Damodaran (1999), the synthetic zero coupon bond has maturity equal to the face-value-weighted average of the McCauley durations of the different issues and a face value which includes all the principal outstanding on the debt and coupons that will become due on existing debt. Therefore the default probability of the liability portfolio is entirely concentrated on the maturity date of the synthetic zero coupon bond instead of being spread among all the actual servicing dates and the resulting corporate valuations reflect such distortion.

In order to show the magnitude of this phenomenon the discrete structural valuation model presented in Cenci–Gheno (2005) is used. This model takes into account the default risk involved with any debt structure which is not fully captured using the Merton model in its simplest form (i.e. the Black–Scholes formula) when the debt of the firm is not a single zero coupon bond. Considering a firm financed through equity and two issues of zero coupon bonds with face values of €500 and maturity of 5 and 10 years, a risk-free rate of 5%, equity and debt valuations are shown in Table 1 and 2 respectively using the Black–Scholes (1973) formula and the Cenci–Gheno (2005) model with exogenous default.

	V(0)													
	700		800		900		1000		1100		1200		1300	
σ_V	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**
0,1	82,07	6,82%	149,39	6,05%	231,08	5,64%	321,56	5,43%	416,86	5,33%	514,62	5,28%	613,58	5,26%
0,2	156,13	8,78%	222,80	7,86%	297,32	7,20%	377,91	6,72%	463,12	6,37%	551,84	6,11%	643,22	5,91%
0,3	227,52	11,01%	297,22	10,01%	372,02	9,24%	450,94	8,63%	533,20	8,41%	618,21	7,74%	705,45	7,41%
0,4	295,03	13,56%	368,48	12,50%	445,54	11,64%	525,56	10,94%	609,04	10,36%	692,59	9,87%	778,87	9,45%
0,5	357,67	16,46%	434,93	15,33%	514,86	14,41%	597,00	13,64%	680,98	12,99%	766,53	12,42%	853,41	11,93%
0,6	414,71	19,76%	495,61	18,57%	578,51	17,58%	663,08	16,74%	749,05	16,02%	836,23	15,40%	924,43	14,84%
0,7	465,70	23,53%	549,92	22,26%	635,66	21,20%	722,67	20,29%	810,76	19,51%	899,76	18,82%	989,57	18,21%

Table 1. Black–Scholes (1973) – Equity values and debt yields for different input of asset volatility and initial asset value

	V(0)													
	700		800		900		1000		1100		1200		1300	
σ_V	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**	*S(0)*	*i**
0,1	63,41	6,38%	133,13	5,68%	218,80	5,37%	312,14	5,23%	409,27	5,17%	508,06	5,14%	607,59	5,13%
0,2	136,15	8,22%	203,27	7,35%	277,53	6,71%	360,22	6,30%	445,95	5,97%	536,97	5,77%	629,55	5,61%
0,3	203,32	10,21%	272,66	9,26%	345,37	8,47%	425,63	7,93%	508,65	7,49%	593,29	7,10%	682,67	6,84%
0,4	267,80	12,47%	335,95	11,37%	412,97	10,52%	492,65	9,87%	576,26	9,37%	659,86	8,88%	746,64	8,51%
0,5	326,64	14,95%	399,15	13,73%	476,97	12,83%	556,97	12,06%	641,89	11,51%	727,26	11,00%	812,64	10,51%
0,6	380,32	17,68%	457,01	16,43%	537,22	15,44%	618,39	14,57%	702,76	13,88%	790,28	13,36%	877,81	12,86%
0,7	428,83	20,71%	509,57	19,43%	592,27	18,37%	674,98	17,38%	759,08	16,53%	847,32	15,94%	937,01	15,43%

Table 2. Cenci–Gheno (2005) – Equity values and debt yields for different input of asset volatility and initial asset value

As shown in Figure 1 the misspecification of the debt servicing, which corresponds to a reduction of the credit risk, leads to overestimates of the equity value. In general for a given level of asset volatility the higher the leverage the higher the overestimate.

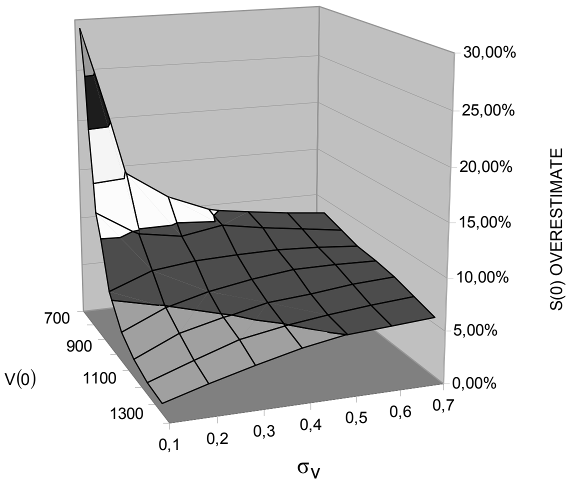

Figure 1. Overestimate of equity value (Black–Scholes (1973) vs. Cenci–Gheno (2005)).

V. Final Remarks

The Merton model is definitely a powerful and versatile model. As with any other financial model in order to implement it properly its assumptions must be taken carefully into consideration. With this in mind in section II and III some issues about asset value and volatility estimations have been considered. Finally in section IV it has been shown how valuations of firms with complex debt structures are distorted when unrealistic simplifications are made.

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