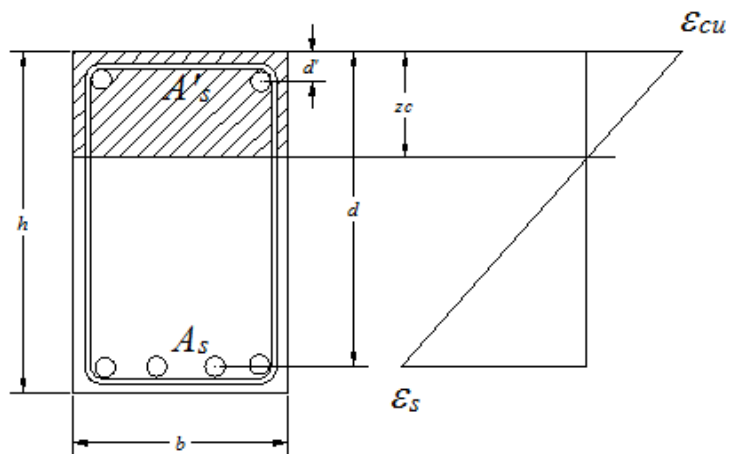
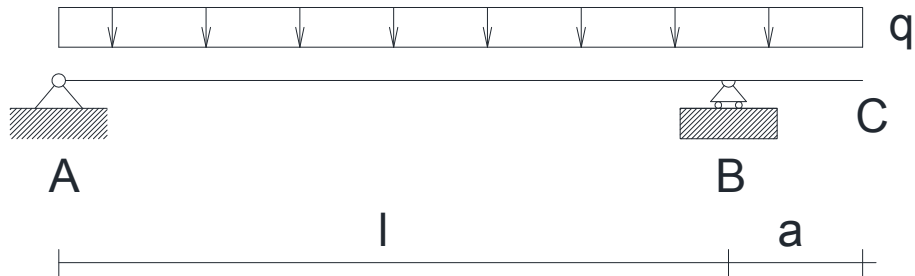


## Cemento armato: flessione, esercizio n.2



Per la trave in figura, di luce  $l := 5.0 \cdot \text{m}$ ,  $a := 1.5 \cdot \text{m}$ , soggetta ad un carico di esercizio  $q := 65 \cdot \frac{\text{kN}}{\text{m}}$  con una sezione rettangolare di altezza  $h := 50 \cdot \text{cm}$  e base  $b := 25 \cdot \text{cm}$  e la seguente armatura:

campata: armatura tesa di  $n_{sc} := 4$  barre  $\phi_c := 20 \cdot \text{mm}$  e armatura compressa di  $n_{sc1} := 2$  barre  $\phi_{c1} := 16 \cdot \text{mm}$ ;

appoggio B:  $n_{sa} := 4$  barre  $\phi_a := 16 \cdot \text{mm}$  e armatura compressa di  $n_{sa1} := 2$  barre  $\phi_{a1} := 20 \cdot \text{mm}$  distanza dai bordi  $d_1 := 4 \cdot \text{cm}$ ;

calcolare:

- 1) le sollecitazioni massime,
- 2) le tensioni nel calcestruzzo e nell'acciaio prodotte dal momento massimo negativo e positivo,
- 3) il momento ultimo delle sezioni di appoggio e campata

Calcestruzzo:  
Acciaio  $E_s := 200000 \cdot \text{MPa}$

$$f_{ck} := 27 \text{ MPa} \quad f_{cd} := \frac{0.85 f_{ck}}{1.5} = 15.3 \cdot \text{MPa} \quad \epsilon_{cu} := 3.5 \cdot 10^{-3}$$

$$f_{yk} := 450 \text{ MPa} \quad f_{yd} := \frac{f_{yk}}{1.15} = 391.304 \cdot \text{MPa}$$

$$\epsilon_{yd} := \frac{f_{yd}}{E_s} = 1.957 \times 10^{-3}$$

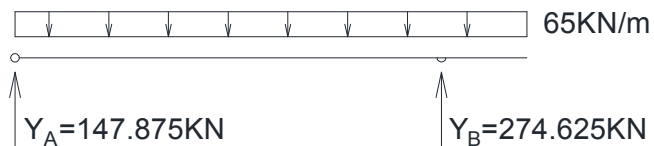
$$\sigma_c := 0.6 \cdot f_{ck} = 16.2 \cdot \text{MPa}$$

$$\sigma_s := 0.8 \cdot f_{yk} = 360 \cdot \text{MPa}$$

### **Calcolo delle reazioni vincolari**

$$Y_B := q \cdot \frac{(1+a)^2}{2l} \quad Y_B = 274.625 \cdot \text{kN}$$

$$Y_A := q \cdot (1+a) - Y_B \quad Y_A = 147.875 \cdot \text{kN}$$



### **Calcolo delle sollecitazioni**

#### **Taglio**

$$V_A := Y_A \quad V_A = 147.875 \cdot \text{kN}$$

$$V(x) := Y_A - q \cdot x$$

$$x_0 := \frac{Y_A}{q} \quad x_0 = 2.275 \text{ m}$$

$$V(l) = -177.125 \cdot \text{kN}$$

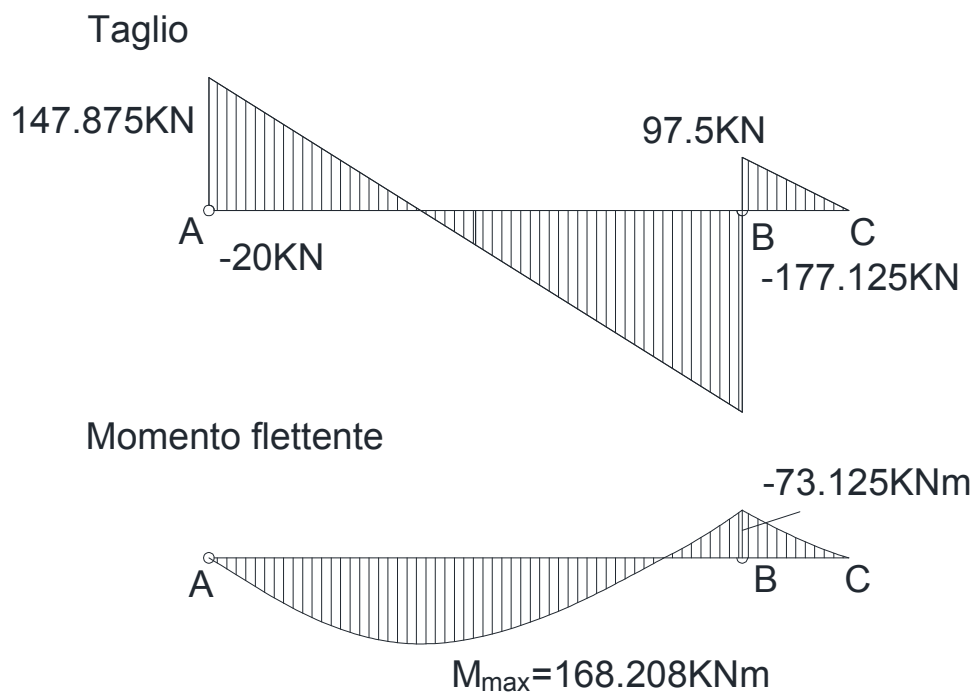
$$V_B := Y_B - |V(l)| \quad V_B = 97.5 \cdot \text{kN}$$

### Momento

$$M(x) := V_A \cdot x - q \cdot \frac{x^2}{2}$$

$$M_{\max} := M(x_0) = 168.208 \cdot \text{kN} \cdot \text{m}$$

$$M(l) = -73.125 \cdot \text{kN} \cdot \text{m}$$



### Verifica in campata

#### Armature

$$A_{sc} := n_{sc} \cdot \pi \cdot \frac{\phi_c^2}{4}$$

$$A_{sc} = 12.566 \cdot \text{cm}^2$$

$$A_{sc1} := n_{sc1} \cdot \pi \cdot \frac{\phi_{c1}^2}{4} \quad A_{sc1} = 4.021 \cdot \text{cm}^2$$

$$d := h - d_1 \quad d = 46 \cdot \text{cm}$$

$$A_{stc} := A_{sc} + A_{sc1} \quad A_{stc} = 16.588 \cdot \text{cm}^2$$

$$d_{Gc} := \frac{A_{sc} \cdot d + A_{sc1} \cdot d_1}{A_{stc}} \quad d_{Gc} = 35.818 \cdot \text{cm}$$

$$z_c := \left( \sqrt{1 + \frac{2 \cdot b \cdot d_{Gc}}{n \cdot A_{stc}}} - 1 \right) \cdot \frac{n \cdot A_{stc}}{b} \quad z_c = 18.543 \cdot \text{cm}$$

$$J_n := \frac{b \cdot z_c^3}{3} + n \cdot \left[ A_{sc1} \cdot (z_c - d_1)^2 + A_{sc} \cdot (z_c - d)^2 \right] \quad J_n = 2.08 \times 10^5 \cdot \text{cm}^4$$

$$\sigma_{ccampata} := \frac{M_{\max}}{J_n} \cdot z_c \quad \sigma_{ccampata} = 14.996 \cdot \text{MPa}$$

$$\sigma_{scampata} := n \cdot \frac{M_{\max}}{J_n} \cdot (d - z_c) \quad \sigma_{scampata} = 333.069 \cdot \text{MPa}$$

## Verifica all'appoggio in B

### Armature

$$A_{sa} := n_{sa} \cdot \pi \cdot \frac{\phi_a^2}{4} \quad A_{sa} = 8.042 \cdot \text{cm}^2$$

$$A_{sa1} := n_{sa1} \cdot \pi \cdot \frac{\phi_{a1}^2}{4} \quad A_{sa1} = 6.283 \cdot \text{cm}^2$$

$$A_{sta} := A_{sa} + A_{sa1} \quad A_{sta} = 14.326 \cdot \text{cm}^2$$

$$d_{Ga} := \frac{A_{sa} \cdot d + A_{sa1} \cdot d_1}{A_{sta}} \quad d_{Ga} = 27.579 \cdot \text{cm}$$

$$z_a := \left( \sqrt{1 + \frac{2 \cdot b \cdot d_{Ga}}{n \cdot A_{sta}}} - 1 \right) \cdot \frac{n \cdot A_{sta}}{b} \quad z_a = 14.814 \cdot \text{cm}$$

$$J_{na} := \frac{b \cdot z_a^3}{3} + n \cdot \left[ A_{sal} \cdot (z_a - d_1)^2 + A_{sa} \cdot (z_a - d)^2 \right] \quad J_{na} = 1.554 \times 10^5 \cdot \text{cm}^4$$

$$\sigma_{cappoggio} := \frac{|M(l)|}{J_{na}} \cdot z_a \quad \sigma_{cappoggio} = 6.969 \cdot \text{MPa}$$

$$\sigma_{sappoggio} := n \cdot \frac{|M(l)|}{J_{na}} \cdot (d - z_a) \quad \sigma_{sappoggio} = 220.068 \cdot \text{MPa}$$

### **Calcolo del momento ultimo in campata**

armatura bilanciata

$$z_{cb} := d \cdot \frac{\epsilon_{cu}}{(\epsilon_{cu} + \epsilon_{yd})} \quad \epsilon_{s1b} := \epsilon_{cu} \cdot \frac{(z_{cb} - d_1)}{z_{cb}}$$

$$z_{cb} = 0.295 \text{ m} \quad \epsilon_{s1b} = 3.026 \times 10^{-3}$$

$$A_{sb} := \frac{(0.81 \cdot f_{cd} \cdot b \cdot z_{cb})}{f_{yd}} + A_{sc1} \cdot \frac{\min(\epsilon_{s1b} \cdot E_s, f_{yd})}{f_{yd}}$$

$$A_{sb} = 27.383 \cdot \text{cm}^2$$

$$A_{sc} < A_{sb}$$

$$z_{cu} := \frac{(A_{sc} - A_{sc1}) \cdot f_{yd}}{0.81 \cdot f_{cd} \cdot b} \quad z_{cu} = 0.108 \text{ m}$$

$$\epsilon_{s1u} := \epsilon_{cu} \cdot \frac{(z_{cu} - d_1)}{z_{cu}} \quad \epsilon_{s1u} = 2.203 \times 10^{-3}$$

$$M_u := A_{sc} \cdot f_{yd} \cdot (d - 0.4 \cdot z_{cu}) + A_{sc1} \cdot f_{yd} \cdot (0.4 \cdot z_{cu} - d_1) \quad M_u = 205.466 \cdot \text{kN} \cdot \text{m}$$

$$M_{u1} := 0.9 \cdot d \cdot A_{sc} \cdot f_{yd} = 203.575 \cdot \text{kN} \cdot \text{m}$$

## Calcolo del momento ultimo all'appoggio

armatura bilanciata

$$A_{sbapp} := \frac{(0.81 \cdot f_{cd} \cdot b \cdot z_{cb})}{f_{yd}} + A_{sa1} \cdot \frac{\min(\epsilon_{s1b} \cdot E_s, f_{yd})}{f_{yd}}$$

$$A_{sbapp} = 29.645 \cdot \text{cm}^2$$

$$A_{sa} < A_{sb}$$

$$z_{cua} := \frac{(A_{sa} - A_{sa1}) \cdot f_{yd}}{0.81 \cdot f_{cd} \cdot b} = 2.222 \cdot \text{cm}$$

$$\beta := \frac{(A_{sa} \cdot f_{yd} - A_{sa1} \cdot \epsilon_{cu} \cdot E_s)}{0.81 \cdot f_{cd} \cdot b} = -4.038 \cdot \text{cm}$$

$$\gamma := \frac{(A_{sa} \cdot \epsilon_{cu} \cdot d_1 \cdot E_s)}{0.81 \cdot f_{cd} \cdot b} = 72.683 \cdot \text{cm}^2$$

$$z_{cuv} := \frac{\beta + \sqrt{\beta^2 + 4 \cdot \gamma}}{2} = 6.742 \cdot \text{cm}$$

$$\epsilon_{s1} := \epsilon_{cu} \cdot \frac{(z_c - d_1)}{z_c} = 1.423 \times 10^{-3}$$

$$M_u := A_{sa} \cdot f_{yd} \cdot (d - 0.42 \cdot z_c) - A_{sa1} \cdot \epsilon_{s1} \cdot E_s \cdot (d_1 - 0.42 \cdot z_c) = 133.763 \cdot \text{kN} \cdot \text{m}$$

