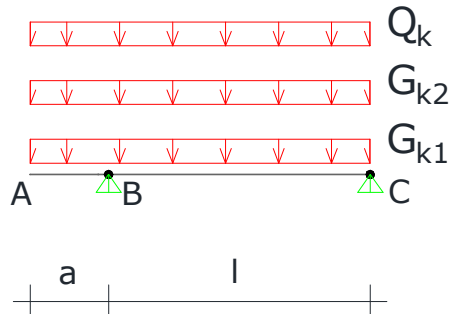


Cemento armato: flessione 19 12 2011



Per la struttura in figura, determinare le sollecitazioni N,V,M.

1. Progettare allo SLU le armature di flessione della trave.
2. Eseguire le verifiche agli SLE (tensioni ed apertura delle fessure).

Dati

Dimensioni

$$l := 4.5 \cdot \text{m} \quad a := 1.5 \cdot \text{m}$$

Carichi (valori caratteristici)

$$\text{Carichi permanenti } G_{k1} := 17 \cdot \frac{\text{kN}}{\text{m}} \quad G_{k2} := 15 \cdot \frac{\text{kN}}{\text{m}}$$

$$\text{Carico variabile } Q_k := 10 \cdot \frac{\text{kN}}{\text{m}}$$

Trave: sezione rettangolare di base $B := 25 \cdot \text{cm}$ altezza $H := 45 \cdot \text{cm}$

copriferro $d_1 := 3 \cdot \text{cm}$

Materiali

Acciaio $f_{yk} := 450 \cdot \text{MPa}$ $E_s := 200000 \cdot \text{MPa}$ $n := 15$

Calcestruzzo $f_{ck} := 30 \cdot \text{MPa}$ $\epsilon_{cu} := 0.0035$

Coefficienti di sicurezza (SLU)

Carichi	$\gamma_{G1} := 1.3$	$\gamma_{G2} := 1.5$	$\gamma_{Q2} := 1.5$
Acciaio c.a.	$\gamma_s := 1.15$		
Calcestruzzo	$\gamma_c := 1.5$		

Coefficienti di combinazione

$$\psi_{11} := 0.5 \quad \psi_{21} := 0.3$$

Per il progetto e la verifica a flessione allo SLU della struttura in cemento armato la resistenza di progetto del calcestruzzo deve essere moltiplicata per 0.85.

Carico uniformemente per le diverse combinazioni

SLU	$q_{SLU} := 1.3 \cdot G_{k1} + 1.5 \cdot G_{k2} + 1.5 \cdot Q_k$	$q_{SLU} = 59.6 \frac{\text{kN}}{\text{m}}$
Comb. rara	$q_{rara} := G_{k1} + G_{k2} + Q_k$	$q_{rara} = 42 \frac{\text{kN}}{\text{m}}$
Comb. frequente	$q_{freq} := G_{k1} + G_{k2} + \psi_{11} \cdot Q_k$	$q_{freq} = 37 \frac{\text{kN}}{\text{m}}$
Comb. quasi permanente	$q_{qperm} := G_{k1} + G_{k2} + \psi_{21} \cdot Q_k$	$q_{qperm} = 35 \frac{\text{kN}}{\text{m}}$

Reazioni vincolari e sollecitazioni

$$q_d := \begin{pmatrix} q_{SLU} \\ q_{rara} \\ q_{freq} \\ q_{qperm} \end{pmatrix}$$

$$\text{Reazioni vincolari} \quad Y_B := \frac{1}{l} \cdot \left[q_d \cdot \frac{(l+a)^2}{2} \right] \quad Y_B = \begin{pmatrix} 238.4 \\ 168 \\ 148 \\ 140 \end{pmatrix} \cdot \text{kN}$$

$$Y_A := q_d \cdot (l + a) - Y_B \quad Y_A = \begin{pmatrix} 119.2 \\ 84 \\ 74 \\ 70 \end{pmatrix} \cdot \text{kN}$$

Sollecitazioni

Taglio

$$V_d(x) := \text{if}(x \leq l, Y_A - q_d \cdot x, Y_A + Y_B - q_d \cdot x)$$

Taglio massimo: $V_A := V_d(0)$ $V_A = \begin{pmatrix} 119.2 \\ 84 \\ 74 \\ 70 \end{pmatrix} \cdot \text{kN}$

$$V_{Bd} := V_d(1 \cdot 0.999999) \quad V_{Bd} = \begin{pmatrix} -149 \\ -105 \\ -92.5 \\ -87.5 \end{pmatrix} \cdot \text{kN}$$

$$V_{Bs} := V_d(1 \cdot 1.000001) \quad V_{Bs} = \begin{pmatrix} 89.4 \\ 63 \\ 55.5 \\ 52.5 \end{pmatrix} \cdot \text{kN}$$

$$V_C := V_d(l + a) \quad V_C = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \text{kN}$$

Momento flettente

$$M_d(x) := \text{if}\left[x \leq l, Y_A \cdot x - q_d \cdot \frac{x^2}{2}, M_d(l) + V_{Bs} \cdot (x - l) - q_d \cdot \frac{(x - l)^2}{2}\right]$$

$$M_B := M_d(l) \quad M_B = \begin{pmatrix} -67.05 \\ -47.25 \\ -41.625 \\ -39.375 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$M_C := M_d(l + a)$$

$$M_C = \begin{pmatrix} -4.023 \times 10^{-4} \\ -2.835 \times 10^{-4} \\ -2.497 \times 10^{-4} \\ -2.362 \times 10^{-4} \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

Momento Massimo

$$x_{\max} := \frac{Y_{A_1}}{q_{d_1}}$$

$$x_{\max} = 2 \frac{\text{m}^2 \cdot \text{s}^2}{\text{kg}} \text{ Pa}$$

$$M_{\max} := M_d(x_{\max})$$

$$M_{\max} = \begin{pmatrix} 119.2 \\ 84 \\ 74 \\ 70 \end{pmatrix} \cdot \text{kN} \cdot \text{m}$$

$$M_{\max\text{SLU}} := M_{\max_0}$$

$$M_{\max\text{SLU}} = 119.2 \text{ kN} \cdot \text{m}$$

$$M_{\max\text{rara}} := M_{\max_1}$$

$$M_{\max\text{rara}} = 84 \text{ kN} \cdot \text{m}$$

$$M_{\max\text{freq}} := M_{\max_2}$$

$$M_{\max\text{freq}} = 74 \text{ kN} \cdot \text{m}$$

$$M_{\max\text{qperm}} := M_{\max_3}$$

$$M_{\max\text{qperm}} = 70 \text{ kN} \cdot \text{m}$$

$$M_{\text{BSLU}} := M_{B_0}$$

$$M_{\text{BSLU}} = -67.05 \text{ kN} \cdot \text{m}$$

$$M_{\text{Brara}} := M_{B_1}$$

$$M_{\text{Brara}} = -47.25 \text{ kN} \cdot \text{m}$$

$$M_{\text{Bfreq}} := M_{B_2}$$

$$M_{\text{Bfreq}} = -41.625 \text{ kN} \cdot \text{m}$$

$$M_{\text{Bqperm}} := M_{B_3}$$

$$M_{\text{Bqperm}} = -39.375 \text{ kN} \cdot \text{m}$$

Progetto allo SLU delle armature della trave in cemento armato

sezione di momento massimo in campata:

$$f_{yd} := \frac{f_{yk}}{\gamma_s} \quad f_{yd} = 391.304 \cdot \text{MPa} \quad f_{cd} := \frac{0.85f_{ck}}{\gamma_c} \quad f_{cd} = 17 \cdot \text{MPa}$$

$$d := H - d_1 = 42 \cdot \text{cm}$$

$$A_{s\text{cmin}} := \frac{|M_{\text{maxSLU}}|}{0.9 \cdot d \cdot f_{yd}} = 8.059 \cdot \text{cm}^2$$

$$\phi_s := 16 \text{mm}$$

$$n_{sc} := \text{ceil} \left(\frac{4A_{s\text{cmin}}}{\pi \cdot \phi_s^2} \right) = 5$$

$$A_{sc} := n_{sc} \cdot \pi \cdot \left(\frac{\phi_s}{2} \right)^2 = 10.053 \cdot \text{cm}^2$$

sezione di momento negativo all'appoggio in B:

$$A_{s\text{Bmin}} := \frac{|M_{\text{BSLU}}|}{0.9 \cdot d \cdot f_{yd}} = 4.533 \cdot \text{cm}^2$$

$$\phi_{s1} := 16 \text{mm}$$

$$n_{sB} := \text{ceil} \left(\frac{4A_{s\text{Bmin}}}{\pi \cdot \phi_s^2} \right) = 3$$

$$n_{sB} := \max(n_{sB}, 2) \quad n_{sB} = 3$$

$$A_{sB} := n_{sB} \cdot \pi \cdot \frac{\phi_{s1}^2}{4} = 6.032 \cdot \text{cm}^2$$

Calcolo del momento ultimo in campata

armatura bilanciata

$$n_{s1c} := 2 \quad A_{s1c} := n_{s1c} \cdot \pi \cdot \frac{\phi_{s1}^2}{4} = 4.021 \cdot \text{cm}^2$$

$$\varepsilon_{yd} := \frac{f_{yd}}{E_s} \quad \varepsilon_{yd} = 1.957 \times 10^{-3}$$

$$\sigma_s(\varepsilon) := \text{if} \left(|\varepsilon| < \frac{f_{yd}}{E_s}, E_s \cdot \varepsilon, f_{yd} \cdot \text{sign}(\varepsilon) \right)$$

$$z_b := d \cdot \frac{\varepsilon_{cu}}{(\varepsilon_{cu} + \varepsilon_{yd})} = 26.94 \cdot \text{cm} \quad \varepsilon_{s1b} := \varepsilon_{cu} \cdot \frac{(z_b - d_1)}{z_b} = 3.11 \times 10^{-3}$$

$$A_{sb} := \frac{(0.81 f_{cd} \cdot B \cdot z_b)}{f_{yd}} + A_{s1c} = 27.722 \cdot \text{cm}^2 \quad A_{sc} = 10.053 \cdot \text{cm}^2 < A_{sb}$$

Calcolo della posizione dell'asse neutro per $A_s < A_{sb}$ e $\varepsilon_{s1} > \varepsilon_{yd}$

$$z_{c1} := \frac{(A_{sc} - A_{s1c}) \cdot f_{yd}}{0.81 f_{cd} \cdot B} = 6.856 \cdot \text{cm}$$

$$\varepsilon_{s1} := \varepsilon_{cu} \cdot \frac{(z_{c1} - d_1)}{z_{c1}} = 1.969 \times 10^{-3} \quad \text{isw} := \varepsilon_{s1} < \varepsilon_{yd} = 0$$

Calcolo della posizione dell'asse neutro per per $A_s < A_{sb}$ e $\varepsilon_{s1} < \varepsilon_{yd}$

$$\beta := \frac{(A_{sc} \cdot f_{yd} - A_{s1c} \cdot \varepsilon_{cu} \cdot E_s)}{0.81 f_{cd} \cdot B} = 3.25 \cdot \text{cm}$$

$$\gamma := \frac{(A_{s1c} \cdot \varepsilon_{cu} \cdot d_1 \cdot E_s)}{0.81 f_{cd} \cdot B} = 24.53 \cdot \text{cm}^2$$

$$z_{c2} := \frac{\beta + \sqrt{\beta^2 + 4 \cdot \gamma}}{2} = 6.838 \cdot \text{cm}$$

Posizione dell'asse neutro

$$z_c := \text{if}(\text{isw} = 0, z_{c1}, z_{c2}) = 6.856 \cdot \text{cm}$$

$$\varepsilon_{s1} := \varepsilon_{cu} \cdot \frac{(z_c - d_1)}{z_c} = 1.969 \times 10^{-3} \quad \sigma_{s1} := \min(E_s \cdot \varepsilon_{s1}, f_{yd})$$

$$M_u := A_{sc} \cdot f_{yd} \cdot (d - 0.42 \cdot z_c) - A_{s1c} \cdot \sigma_{s1} \cdot (d_1 - 0.42 \cdot z_c) = 153.703 \cdot \text{kN} \cdot \text{m} \\ > M_{\text{maxSLU}} = 119.2 \cdot \text{kN} \cdot \text{m}$$

Verifica delle tensioni in campata

$$A_{\text{stot}} := A_{sc} + A_{s1c} \quad A_{\text{stot}} = 14.074 \cdot \text{cm}^2$$

$$d_G := \frac{A_{sc} \cdot d + A_{s1c} \cdot d_1}{A_{\text{stot}}} \quad d_G = 30.857 \cdot \text{cm}$$

$$z_c := \left(\sqrt{1 + \frac{2 \cdot B \cdot d_G}{n \cdot A_{\text{stot}}}} - 1 \right) \cdot \frac{n \cdot A_{\text{stot}}}{B} \quad z_c = 15.896 \cdot \text{cm}$$

$$J_n := \frac{B \cdot z_c^3}{3} + n \cdot [A_{s1c} \cdot (z_c - d_1)^2 + A_{sc} \cdot (z_c - d)^2] \quad J_n = 1.463 \times 10^5 \cdot \text{cm}^4$$

$$\sigma_{\text{crara}} := \frac{M_{\text{maxrara}}}{J_n} \cdot z_c \quad \sigma_{\text{crara}} = 9.129 \cdot \text{MPa} < 0.6 \cdot f_{ck} = 18 \text{ MPa}$$

$$\sigma_{\text{cqperm}} := \frac{M_{\text{maxqperm}}}{J_n} \cdot z_c \quad \sigma_{\text{cqperm}} = 7.608 \cdot \text{MPa} < 0.45 \cdot f_{ck} = 13.5 \text{ MPa}$$

$$\sigma_{\text{srara}} := n \cdot \frac{M_{\text{maxrara}}}{J_n} \cdot (d - z_c) \quad \sigma_{\text{srara}} = 224.882 \cdot \text{MPa} < 0.8 \cdot f_{yk} = 360 \text{ MPa}$$

Calcolo del momento ultimo all'appoggio in B

$$n_{s1B} := 2 \quad A_{s1B} := n_{s1B} \cdot \pi \cdot \frac{\phi_s^2}{4} = 4.021 \cdot \text{cm}^2$$

armatura bilanciata

$$A_{sbB} := \frac{(0.81 f_{cd} \cdot B \cdot z_b)}{f_{yd}} + A_{s1B} = 27.722 \cdot \text{cm}^2 \quad A_{sB} = 6.032 \cdot \text{cm}^2 < A_{sb}$$

Calcolo della posizione dell'asse neutro per $A_s < A_{sb}$ e $\epsilon_{s1} > \epsilon_{yd}$

$$z_{c1} := \frac{(A_{sB} - A_{s1B}) \cdot f_{yd}}{0.81 f_{cd} \cdot B} = 2.285 \cdot \text{cm}$$

$$\epsilon_{s1} := \epsilon_{cu} \cdot \frac{(z_{c1} - d_1)}{z_{c1}} = -1.094 \times 10^{-3} \quad isw := \epsilon_{s1} < \epsilon_{yd} = 1$$

Calcolo della posizione dell'asse neutro per per $A_s < A_{sb}$ e $\epsilon_{s1} < \epsilon_{yd}$

$$\beta := \frac{(A_{sB} \cdot f_{yd} - A_{s1B} \cdot \epsilon_{cu} \cdot E_s)}{0.81 f_{cd} \cdot B} = -1.32 \cdot \text{cm}$$

$$\gamma := \frac{(A_{s1B} \cdot \epsilon_{cu} \cdot d_1 \cdot E_s)}{0.81 f_{cd} \cdot B} = 24.53 \cdot \text{cm}^2$$

$$z_{c2} := \frac{\beta + \sqrt{\beta^2 + 4 \cdot \gamma}}{2} = 4.336 \cdot \text{cm}$$

Posizione dell'asse neutro

$$z_c := \text{if}(isw = 0, z_{c1}, z_{c2}) = 4.336 \cdot \text{cm}$$

$$\epsilon_{s1} := \epsilon_{cu} \cdot \frac{(z_c - d_1)}{z_c} = 1.079 \times 10^{-3} \quad \sigma_{s1} := \min(E_s \cdot \epsilon_{s1}, f_{yd})$$

$$M_u := A_{sB} \cdot f_{yd} \cdot (d - 0.42 \cdot z_c) - A_{s1B} \cdot \sigma_{s1} \cdot (d_1 - 0.42 \cdot z_c) = 93.811 \cdot \text{kN} \cdot \text{m} > |M_{BSLU}| = 67.05 \cdot \text{kN} \cdot \text{m}$$

Verifica delle tensioni all'appoggio in B

$$d_G := \frac{A_{sB} \cdot d + A_{s1B} \cdot d_1}{A_{stot}} \quad d_G = 18.857 \cdot \text{cm}$$

$$z_c := \left(\sqrt{1 + \frac{2 \cdot B \cdot d_G}{n \cdot A_{stot}}} - 1 \right) \cdot \frac{n \cdot A_{stot}}{B} \quad z_c = 11.299 \cdot \text{cm}$$

$$J_n := \frac{B \cdot z_c^3}{3} + n \cdot \left[A_{s1B} \cdot (z_c - d_1)^2 + A_{sB} \cdot (z_c - d)^2 \right] \quad J_n = 1.015 \times 10^5 \cdot \text{cm}^4$$

$$\sigma_{crara} := \frac{|M_{Brara}|}{J_n} \cdot z_c \quad \sigma_{crara} = 5.262 \cdot \text{MPa} < 0.6 \cdot f_{ck} = 18 \text{ MPa}$$

$$\sigma_{cqperm} := \frac{|M_{Bqperm}|}{J_n} \cdot z_c \quad \sigma_{cqperm} = 4.385 \cdot \text{MPa} < 0.45 \cdot f_{ck} = 13.5 \text{ MPa}$$

$$\sigma_{srara} := n \cdot \frac{|M_{Brara}|}{J_n} \cdot (d - z_c) \quad \sigma_{srara} = 214.474 \cdot \text{MPa} < 0.8 \cdot f_{yk} = 360 \text{ MPa}$$

Verifica dell'ampiezza delle fessure in campata

$$f_{ck} := 30$$

$$E_{cm} := 22000 \left(\frac{f_{ck} + 8}{10} \right)^{0.3} \text{ MPa} \quad E_{cm} = 3.284 \times 10^4 \text{ MPa}$$

$$f_{ctm} := 0.3 \cdot (f_{ck})^{\frac{2}{3}} \text{ MPa} \quad f_{ctm} = 2.896 \text{ MPa}$$

$$\chi_s := \frac{E_s}{E_{cm}} \quad \chi_s = 6.091$$

$$A_{\text{ceff}} := B \cdot \min \left[2.5 \cdot (H - d), \frac{(H - z_c)}{3}, \frac{H}{2} \right]$$

$$2.5 \cdot (H - d) = 7.5 \text{ cm} \quad \frac{(H - z_c)}{3} = 11.234 \text{ cm} \quad \frac{H}{2} = 22.5 \text{ cm}$$

$$\rho_{\text{eff}} := \frac{A_{\text{sc}}}{A_{\text{ceff}}} \quad \rho_{\text{eff}} = 0.054$$

Tabella 4.1.IV – Criteri di scelta dello stato limite di fessurazione

Gruppi di esigenze	Condizioni ambientali	Combinazione di azioni	Armatura			
			Sensibile		Poco sensibile	
			Stato limite	w_d	Stato limite	w_d
a	Ordinarie	frequente	ap. fessure	$\leq w_2$	ap. fessure	$\leq w_3$
		quasi permanente	ap. fessure	$\leq w_1$	ap. fessure	$\leq w_2$
b	Aggressive	frequente	ap. fessure	$\leq w_1$	ap. fessure	$\leq w_2$
		quasi permanente	decompressione	-	ap. fessure	$\leq w_1$
c	Molto aggressive	frequente	formazione fessure	-	ap. fessure	$\leq w_1$
		quasi permanente	decompressione	-	ap. fessure	$\leq w_1$

w_1, w_2, w_3 sono definiti al § 4.1.2.2.4.1, il valore di calcolo w_d , è definito al § 4.1.2.2.4.6.

$$w_1 := 0.2 \text{ mm}$$

$$w_2 := 0.3 \text{ mm}$$

$$w_3 := 0.4 \text{ mm}$$

$k_1 =$	0.8 per barre ad aderenza migliorata 1.6 per barre lisce
$k_2 =$	1 nel caso di trazione semplice 0.5 nel caso di flessione $(\varepsilon_{\text{max}} + \varepsilon_{\text{min}}) / 2\varepsilon_{\text{max}}$ nel caso di trazione non uniforme
$k_3 =$	3.4
$k_4 =$	0.425

Tab. 13.1. Valori delle costanti k nella formula di s_{max}

$$k_1 := 0.8 \quad k_2 := 0.5 \quad k_3 := 3.4 \quad k_4 := 0.425$$

$$c := d_1 - \frac{\phi_s}{2} \quad c = 22 \text{ mm}$$

$$s_{\text{max}} := k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \cdot \frac{\phi_s}{\rho_{\text{eff}}} \quad s_{\text{max}} = 125.531 \text{ mm}$$

$$k_t := 0.4$$

$$\sigma_{\text{sfreq}} := n \cdot \frac{|M_{\text{maxfreq}}|}{J_n} \cdot (d - z_c) \quad \sigma_{\text{sfreq}} = 335.895 \cdot \text{MPa}$$

$$\sigma_{\text{sqperm}} := n \cdot \frac{|M_{\text{maxqperm}}|}{J_n} \cdot (d - z_c) \quad \sigma_{\text{sqperm}} = 317.739 \cdot \text{MPa}$$

$$\epsilon_{\text{smfreq}} := \frac{1}{E_s} \cdot \left[\sigma_{\text{sfreq}} - k_t \cdot \frac{f_{\text{ctm}}}{\rho_{\text{eff}}} \cdot (1 + \chi_s \cdot \rho_{\text{eff}}) \right] \quad \epsilon_{\text{smfreq}} = 1.536 \times 10^{-3}$$

$$\epsilon_{\text{smqperm}} := \frac{1}{E_s} \cdot \left[\sigma_{\text{sqperm}} - k_t \cdot \frac{f_{\text{ctm}}}{\rho_{\text{eff}}} \cdot (1 + \chi_s \cdot \rho_{\text{eff}}) \right] \quad \epsilon_{\text{smqperm}} = 1.445 \times 10^{-3}$$

$$w_{\text{dfreq}} := \epsilon_{\text{smfreq}} \cdot s_{\text{max}} \quad w_{\text{dfreq}} = 0.193 \text{ mm} < w_3 = 0.4 \text{ mm}$$

$$w_{\text{dqperm}} := \epsilon_{\text{smqperm}} \cdot s_{\text{max}} \quad w_{\text{dqperm}} = 0.181 \text{ mm} < w_2 = 0.3 \text{ mm}$$

