ENTRY COSTS AND THE DYNAMICS OF BUSINESS FORMATION

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Entry costs and the dynamics of business formation

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Abstract

This paper studies the implications of entry costs for business formation in a dynamic stochastic general equilibrium model with endogenous entry and exit. The paper first documents some facts about business formation in the US. Exit is more volatile than entry, both are more volatile than output and co-move over the cycle. Firms are less volatile than output and procyclical. Then, it shows that a model with entry and exit can replicate these facts fairly well. In addition it captures important features of the US business cycle, outperforming models with a fixed exit rate and a fixed number of firms. The performance of the model is sensitive to changes in the composition of entry costs.

Keywords: entry costs, firm entry, firm exit, business cycle, business creation, business destruction.

JEL codes: E31; E32; E52

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1 Introduction

A novel line of research has stressed the role of firm entry and creation of new products in propagating business cycle fluctuations.\(^1\) With a few exceptions, that will be discussed later in the paper, this literature overlooks firm exit and the destruction of products as a distinct channel for the transmission of shocks. Some authors, as Bilbiie et al. (2008, 2012) and Ghironi and Méliot (2005), focus on labor costs in the spirit of Grossman and Helpman (1991) and Romer (1990). In these models, start-up activities require labor inputs and entry costs are measured as wages. Others, as Bergin and Corsetti (2008) and Cavallari (2013a, 2013b), assume that investors buy materials for the set-up of a new firm, so that entry costs vary with their price. How to model entry costs is an open question.

Entry costs are akin to investment costs in standard (fixed-variety) business cycle models. As for traditional models, there is some debate on the form of these costs. Specifically, the theoretical discussion is on the composition of investment/entry costs and the extent to which these are subject to nominal frictions. Studies that address the question of the role of nominal frictions in entry models include Bergin and Corsetti (2008), Lewis and Poilly (2012) and Uuksúla (2010), among others. They document a negative relation between nominal interest rate innovations and business creation at odds with the implications of models that measure entry costs as wages. In these models, in fact, a monetary expansion pushes on labour demand, increasing wages and entry costs and discouraging the creation of new firms. For this motive, Bilbiie et al. (2008) consider entry costs fixed in units of consumption in the sticky price version of their model, while Lewis and Poilly (2012) focus on sticky wages and Bergin and Corsetti (2008) measure entry costs as product prices. Successive research has showed that entry costs affect the capacity of these models to replicate some stylized facts of the business cycle.\(^2\) Surprisingly, the process of business formation per se is not included among the facts that need to be explained. Furthermore, these models neglect the role of firm exit for the propagation of shocks.\(^3\)

This paper aims to shed some light on these questions. To this end, it considers an economy that combines a mechanism of firm entry à la Ghironi and Méliot (2005) and endogenous firm exit.


\(^2\) Cavallari (2013a) shows that measuring entry costs in terms of products helps to alleviate the comovement puzzles in international business cycle models. Cavallari (2013b) shows that it helps to overcome the difficulty of entry models in reproducing the smoothness and persistence of macroeconomic variables together with the volatility of profits and markups.

\(^3\) A recent evidence suggests the importance of product destruction for economic activity. Bernard et al. (2010) show that the lost value from product destruction represents 30 percent of US output over a 5-year horizon.
The start-up of a new firm requires a combination of labor and capital. This allows to nest within a unified framework common specifications of investment costs. The exit decision is modelled using stochastic scrap values as is popular in the industrial organization approach: in each period all firms that draw a scrap value of their capital higher than the continuation value will leave the market. These features of the model turn essential to replicate the properties of business formation in the data together with important facts of the US business cycle.

The paper first documents some new facts about business formation in the US, using data at the plant level that cover almost the entire production activity. Exit is more volatile than entry and both are far more volatile than output, almost as much as investments. Entry is pro-cyclical, exit is counter-cyclical and they are negatively correlated between each other. The stock of producers is less volatile than output and pro-cyclical.

Then, it compares the performance of the model at replicating these facts under alternative assumptions on firms’ dynamics and the composition of entry costs. The model with endogenous entry and exit replicates the properties of business formation in the data fairly well. In addition, it matches key moments of the US business cycle at least as well as other business cycle models (actually, it does better at matching the smoothness of consumption, investments and hours and their correlation with output). The model with endogenous exit fares favourably relative to specifications with a fixed exit rate and a fixed number of firms. The reason is the counter-cyclical behaviour of exit, which helps to reduce the excessive smoothness of macroeconomic variables that is typical of standard (fixed variety) business cycle models.

The performance of the model is sensitive to changes in the composition of entry/investment costs. In my setup, these costs affect the process of business formation through both the creation and destruction of new products and firms. The free entry condition in the model links firm value to entry costs. Firm value in turn is inversely related to the probability of exit. In the model, a high volatility of entry reflects a strong incentive to adjust the start-up margin over the cycle, for example by creating a new firm in cyclical upturns. Similarly, the volatility of exit reflects the incentive on the part of incumbents to change market participation, for instance by increasing exits in cyclical downturns. An important contribution of the paper is to clarify the role of endogenous movements in entry costs in shaping firms’ incentives to move in and out of the market. On one extreme, entry costs are strongly pro-cyclical when they are measured as wages. This fact together with pro-cyclical profits generate too smooth entry and too volatile exits. On the opposite extreme, both entry costs and profits move in a counter-cyclical way in the model with entry costs as capital. A higher (and more plausible) volatility of entry comes at the cost of too smooth exits. The best performance is achieved in a setup, like the baseline model in this paper, which replicates pro-cyclical profits as in
the data while at the same time limiting the cyclicality of entry costs.

The paper is organized as follows. Section 2 documents some facts about business creation in the US. Section 3 presents the model in log-linear form. Section 4 discusses its performance at replicating key facts in the data. Section 5 contains conclusive remarks.

2 Business formation

In this section I describe the behaviour of entry and exit over the cycle using US data at the plant level. Entry (exit) is a count of establishments born (shut down) during the last 12 months of each year. Data are annual and cover the period from 1977 to 2011. Macroeconomic data are from the Bureau of Economic Analysis (BEA). Business data are from the Business Dynamics Statistics (BDS) of the US Census Bureau. This dataset covers the practical totality of private non-agricultural activity. It is based on longitudinal data collected at the plant and the firm level. I have checked that the cyclical properties of the series do not depend on the metrics used. The broad coverage of the BDS constitutes a remarkable improvement in the quality of the data, especially for studies like the one in this paper which are centered on the role of business formation for the propagation of aggregate shocks. Because of data availability, previous studies have mainly focused on the manufacturing industry. A non-exhaustive list of examples includes Dunne, Roberts and Samuelson (1988, 1989), who use the Census of Manufactures (CM), Campbell (1998) and more recently Lee and Mukoyama (2012), who use the Annual Survey of Manufactures (ASM). Alternatively, the Survey of Current Business provides economy-wide data on business creation, destruction and net business formation (early examples comprise Chatterjee and Cooper (1993) and Devereux et al. (1996) among others). These series have been discontinued in the 1990s.

In what follows, all variables are logged and HP-filtered with a smoothing parameter of 6.25. Table 1 reports unconditional moments for key variables: the top panel refers to entry, exit, net entry and the stock of establishments; the bottom panel considers output, consumption, investments and hours, all measured in real terms.

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4Output is gross domestic product, consumption is private expenditure, investment is gross investment and employment is total hours. All series are measured at constant prices with base year 2009 and are not seasonally adjusted.

5The BDS dataset is publicly available at http://www.census.gov/ces/dataproducts/bds/. It is part of the confidential Longitudinal Business Database (LBD). It covers most of the country’s economic activity. The only major exclusions are self-employed individuals, employees of private households, railroad employees, agricultural production employees, and most government employees.
Table 1:

A: Business dynamics

<table>
<thead>
<tr>
<th></th>
<th>stand dev</th>
<th>stand dev relative to GDP</th>
<th>corr with GDP</th>
<th>corr with entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>4.96</td>
<td>3.85</td>
<td>0.29</td>
<td>1</td>
</tr>
<tr>
<td>Exit</td>
<td>5.55</td>
<td>4.30</td>
<td>-0.14</td>
<td>-0.18</td>
</tr>
<tr>
<td>Establishments</td>
<td>0.86</td>
<td>0.67</td>
<td>0.55</td>
<td>0.37</td>
</tr>
<tr>
<td>Net Entry</td>
<td>3.51</td>
<td>2.81</td>
<td>0.15</td>
<td>0.40</td>
</tr>
</tbody>
</table>

B: Business cycle

<table>
<thead>
<tr>
<th></th>
<th>stand dev</th>
<th>stand dev relative to GDP</th>
<th>corr with GDP</th>
<th>corr with entry</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.29</td>
<td>1</td>
<td>1</td>
<td>0.19</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.02</td>
<td>0.79</td>
<td>0.89</td>
<td>0.08</td>
</tr>
<tr>
<td>Investment</td>
<td>5.81</td>
<td>4.50</td>
<td>0.92</td>
<td>0.23</td>
</tr>
<tr>
<td>Hours</td>
<td>1.40</td>
<td>1.08</td>
<td>0.88</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Three facts stand out. 1. The process of business creation and destruction is quite volatile. The standard deviation of entry is almost 4 times as high as that of GDP and exit is even more volatile. Not surprisingly, the behaviour of entry and exit is akin to that of investments. As we will see, the similarity with investments goes as far to their role of business cycle amplifiers. 2. Business formation comoves with output. Contemporaneous correlations confirm the pro-cyclical nature of entry and the counter-cyclical nature of exit stressed by previous studies. In addition, dynamic correlations (not shown in table) show that entry comoves especially with output in the previous year (the correlation with past GDP is 0.71) while exit comoves with future output (the correlation with one-year-ahead GDP is -0.58). As a consequence of these dynamics, the stock of establishments moves pro-cyclically. 3. Entry and exit are negatively correlated between each other (the correlation is -0.18) and both are correlated with establishments (the correlation is, respectively, 0.37 and -0.24). Overall, these statistics suggests that there are many producers at the margin of market participation, implying a constant restructuring of US economic activity over the cycle.

The statistics in the bottom panel confirm well-known facts about the US business cycle. Investments are far more volatile than output and strongly pro-cyclical. Consumption is smoother than output while hours are roughly as volatile as output. Both these variables comove with GDP.
3 The economy

The model combines the entry mechanism proposed by Ghironi and Mélitz (2005) with endogenous firm exit and nominal frictions à la Calvo. Entry costs are measured by wages and product prices.

The model is presented in linearized form (an Appendix available upon request describes the non-linear model). Hatted variables denote deviations from a deterministic steady state with zero inflation. Variables without a hat or time subscript refer to the steady state. The equilibrium is symmetric: all households, firms and entrants are identical.

3.1 Households

Households derive utility from consumption $\hat{C}_t$ and disutility from working $\hat{H}_t$ hours for the real wage $\hat{w}_t$. They invest their wealth in a risk-free one-period nominal bond that pays interest rate $\hat{i}_t$ and in shares of a mutual fund of firms that pay dividend income $\hat{d}_t$ and the value of selling the initial share position at the price $\hat{v}_t$. The optimal choice of bonds leads to the Euler equation:

$$ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{i}_t - E_t \pi_{t+1} \right) $$  

(1)

where $\pi_t = \ln(P_t/P_{t-1})$ is the CPI inflation rate, $P_t$ is the welfare-based price index and $\rho > 0$ is the degree of risk aversion. The optimal choice of shares yields an additional Euler equation:

$$ E_t \hat{C}_{t+1} = \hat{C}_t + \hat{v}_t + \frac{1}{\rho} E_t \left( \frac{i + \delta \hat{d}_{t+1}}{1 + i} + \frac{1 - \delta}{1 + \delta} \hat{v}_{t+1} \right) $$  

(2)

where $\delta \in (0, 1)$ denotes the share of firms that exit the market in steady state. The condition above pins down the equity price $\hat{v}_t$.

The optimal trade-off between leisure and work gives the labour supply equation:

$$ \hat{H}_t = -\rho \varphi \hat{C}_t + \varphi \hat{w}_t $$  

(3)

where $\varphi > 0$ is the Frisch elasticity of labour supply.

3.2 Firms

I consider a two-sector economy where capital and/or labor are employed to produce goods and new firms. Let the subscript M denote the manufacturing sector and the subscript E denote the entry sector.

The manufacturing sector is populated by $\hat{N}_t$ firms, each producing a unique variety $j$. Firms
face an identical production function that uses labour as input together with a fixed factor. The presence of fixed production costs implies that not all firms will be able to produce in every period. As will be clear soon, in each period there is a fraction of firms - determined endogenously in the model - that exit the market. The aggregate production function is \( \hat{Y}_t^M = \hat{H}_t^M + \hat{Z}_t + \hat{\varphi}_t \) where \( \hat{Y}_t^M \) is output of the manufacturing sector, \( \hat{\varphi}_t \) is the relative price (the price of a single variety \( p_t(j) \) in terms of \( \pi_t \)), and \( \hat{Z}_t \) denotes labour productivity. Productivity follows an exogenous AR(1) process in logs \( \hat{Z}_t = \rho \hat{Z}_{t-1} + \epsilon_t \) where innovations \( \epsilon_t \) are distributed normally and independently of each other with variance \( \sigma_{\epsilon} \).

Aggregation over varieties takes the love of variety form as in Benassy (1996). The aggregate price is \( P_t = \frac{1}{A^M_t} \left[ \int \int p_t(j)^{1-\theta} \right]^{1/\theta} \) with \( A^M_t \equiv N_t^{\gamma^M-1} \). Here \( \gamma^M > 0 \) denotes the degree of consumers’ love for variety and \( \theta > 1 \) denotes the elasticity of substitution across varieties. \( \gamma^M - 1 \) represents the marginal utility gain from spreading a given amount of consumption on a basket that includes one additional variety in a symmetric equilibrium. An increase in the number of varieties may raise or reduce the aggregate price depending on how much consumers value the benefit of spreading consumption over a larger array of varieties relative to the benefit of consuming more of each variety. The indicator of consumers’ love for variety in log-linear terms is:

\[
\hat{A}^M_t = \left( \gamma^M - \frac{\theta}{(\theta - 1)} \right) \hat{N}_t \quad (4)
\]

Each firm sets the price for its own product facing a downward-looking market demand. Prices are sticky as in Calvo (1983). In each period a firm can set a new price with a fixed probability \( 1 - \alpha \), which is the same across all firms, both incumbents and entrants, and is independent of the time elapsed since the last price change. The optimal price is an average of the desired markup on marginal costs (equal to \( \frac{\theta}{\pi_t - 1} \)) and of the expected future price with weights equal to, respectively, \( (1 - \alpha \frac{(1-\delta)}{1+i}) \) and \( \alpha \frac{(1-\delta)}{1+i} \). The product price \( \hat{\varphi}_t \), the CPI inflation rate \( \pi_t \) and the ex-post markup \( \hat{\varphi}_t \) are given by:

\[
\hat{\varphi}_t = (\theta - 1) \hat{A}^M_t + \frac{\alpha}{1 - \alpha} \pi_t + \frac{1}{(1 - \alpha)(\theta - 1)} \hat{N}_t - \frac{\alpha}{(1 - \alpha)(\theta - 1)} \hat{N}_{t-1} \quad (5)
\]

\[
\pi_t = \zeta \left[ (\theta - 1) \hat{A}^M_t + \left( \rho + \frac{1}{\varphi} \right) \hat{C}_t - \frac{1}{(1 - \alpha)(\theta - 1)} \hat{N}_t - \frac{(1 + \varphi)}{\varphi} \hat{Z}_t + \frac{\alpha}{(1 - \alpha)(\theta - 1)} \hat{N}_{t-1} \right] + \frac{(1 - \delta)}{1+i} E_{t+1} \pi_{t+1} \quad (6)
\]
\[
\hat{\mu}_t = \alpha \frac{(1 - \delta)}{1 + i} (E_t \hat{\omega}_{t+1} - \bar{\omega}_t + \hat{Z}_t)
\]  
\[\zeta = \frac{(1 - \alpha)}{\alpha \psi (1 - \alpha)} \]  

with \( \zeta = \frac{(1 - \alpha)}{\alpha \psi (1 - \alpha)} \).

The start-up sector uses labour and products (capital) to produce a finite mass of entrants \( \hat{N}_t^E \) in each period. The aggregate production function for new firms is Cobb-Douglas \( \hat{N}_t^E = \epsilon \hat{H}_t + (1 - \epsilon) \hat{\nu}_t^E \) with \( \epsilon \in (0, 1) \). Aggregation of products used for start-up investments takes the love of variety form as for consumption. The price of capital is therefore \( P_t^E = \frac{1}{A_t^E} \left[ \int_0^{N_t} p_t(j)^{1 - \theta} dj \right] \frac{1}{1 + \gamma} \) with \( A_t^E \equiv N_t^{\gamma_E} \). Here, \( A_t^E \) is an indicator of efficiency of investments analogous to the love of variety for consumers. \( \hat{N}_t^E \) is given by an equation analogous to (4) where \( \gamma^E \) replaces \( \gamma^M \). The price of capital in units of consumption, \( \frac{P_t^E}{P_t} \), is \( \hat{P}_t^E = \hat{A}_t^M - \hat{A}_t^E \). An increase in the number of firms reduces (increases) this price if \( \gamma^E > \gamma^M \) (\( \gamma^E < \gamma^M \)).

As in Ghironi and Mézit (2005), entrants pay a sunk entry cost in period \( t \) in order to be able to produce in period \( t+1 \). The fact that first-time market participation implies sunk costs is amply documented. In equilibrium, entry takes place up to the point where entry costs equal firm value:

\[
\hat{\nu}_t = \epsilon (\bar{\omega}_t - \hat{Z}_t) + (1 - \epsilon) \hat{P}_t^E
\]  

In this setting, \( \epsilon = 1 \) nests the specification of entry costs as effective labour unions found in Bilbiie et al. (2012), Auray et al. (2012) and Cavallari (2007). With \( \epsilon = 0 \) entry costs are in terms of capital as in Bergin and Corsetti (2008), Cavallari (2013a) and Arespa (2012)). With \( \gamma^M = \gamma^E \) and \( \epsilon = 0 \) entry costs are constant in units of consumption, a case examined by Auray and Eyquem (2011), Bilbiie et al. (2008) and Cavallari (2013b).

I consider an endogenous exit rate. Previous studies have assumed that exit occurs at no cost on a period-by-period basis, as for instance Totzek (2009) and La Croce and Rossi (2014). In these models, firms compare the value of staying - defined as the discounted value of current and future profits and coinciding with the share price - with the value of exiting, which is zero by construction. There is a threshold idiosyncratic productivity such that when productivity is higher than the threshold the firm stays; if productivity is lower, the firm exits. Since firm size and productivity have a one-to-one relation, this means a 100% exit rate for firms smaller than a certain threshold and zero for large firms. However, this stands in contrast to the data. Lee and Mukoyama (2012) document that even large plants with more than 250 employees have an exit rate of over 1% : although exit rates are

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\(^6\)Many studies infer the existence of these costs from the persistence in market participation patterns. Examples include, among others, Roberts and Tybout (1997), Campa (2004), Bernard and Jensen (2004), and Bernard and Wagner (2001)). Das, Roberts and Tybout (2007) provide a method to pin down the magnitude of entry costs based on their non-linear effects on market participation.
indeed higher in small firms, some large firms also exit and some small firms also stay. Here, I take an alternate route. I borrow from the industrial organization approach the notion that a firm will leave the market if the scrap value of its capital is higher than the continuation value.\footnote{Modelling the exit decision using stochastic scrap values is popular in the industrial organization literature (see Doraszelski and Pakes (2007), Weintraub, Benkard and van Roy (2008)).} This has the advantage of implying exit rates between zero and one for firms of any size. Specifically, I assume scrap values $S_j$ to be Pareto distributed across firms. In each period the fraction of firms that leave the market is $\delta_t \equiv \Pr(S_j > \nu_t) = 1 - \Gamma(\nu_t)$ where $\Gamma = 1 - \left( \frac{S_j}{S_{\text{min}}} \right)^{-\xi}$ is the cumulative distribution function of $S_j$, $\xi$ controls the curvature of the distribution and $S_{\text{min}}$ controls the level. All firms with a scrap value draw higher than the continuation value (the share price) will leave the market. For the property of the Pareto distribution, a small fraction of firms will exit after a large rise in the scrap value (or a large fall in firm value).

Finally, the timing of entry and exit implies the following law of motion for firms:

$$\hat{N}_t = (1 - \delta) \hat{N}_{t-1} + \delta \hat{N}_{t-1}^E - \frac{\delta}{\delta - \hat{\delta}_t}$$

where $\hat{\delta}_t = -\xi \hat{\nu}_t$.

### 3.3 Aggregate accounting

GDP is the sum of output of the manufacturing sector and output of the start-up sector $\hat{Y}_t = \frac{Y}{M} \hat{Y}_t^M + \frac{\nu N^E}{Y} (\hat{\nu}_t + \hat{N}_t^E)$. Aggregate accounting implies that output is distributed between labour and profit incomes, yielding:

$$\hat{Y}_t = \frac{wH}{Y} (\hat{\nu}_t + \hat{H}_t) + \frac{dN}{Y} (\hat{d}_t + \hat{N}_t) \quad (9)$$

Equilibrium in the goods and labour markets is given by, respectively:

$$\hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{\nu N^E}{Y} (\hat{N}_t^E - \hat{\nu}_t) \quad (10)$$

$$\hat{H}_t = \frac{H^M}{H} \hat{H}_t^M + \frac{H^E}{H} \hat{H}_t^E$$

The model is closed by specifying the monetary policy in place. I consider the Taylor rule $\hat{\nu}_t = \phi_t \hat{\nu}_{t-1} + \phi_y \pi_t + \phi_y^E \hat{\nu}_t$ (Taylor (1983)). The presence of interest smoothing reflects the need to reduce swings in interest rates in an environment characterized by long and variable lags in the
transmission of monetary policy.

4 Numerical analysis

This section simulates the model using first-order perturbation methods. I start with a description of the mechanics of the model in the wake of a productivity shock. Then, I illustrate the model’s implications for firms’ entry and exit and compare them with empirical evidence.

4.1 Calibration

The calibration refers to the United States. Periods are interpreted as years. Consistent with most macroeconomic studies, the interest rate is \( i = 0.04 \). There is no direct information on the parameters that characterize the distribution of scrap values. The level parameter has no consequences for the dynamics of the model and can be normalized to one without loss of generality. The shape parameter is chosen to reproduce the standard deviation of the firm exit rate, implying \( \kappa = 2.1 \).\(^8\) In calibrations with a fixed exit rate, the probability of exit is set at \( \delta = 0.10 \) to match the empirical level of job destruction per year as in Bilbiie, Ghironi and Méritz (2012).

For ease of comparison with business cycle studies that consider endogenous entry, the parameterization of consumers’ preferences is based on the workhorse model of Bilbiie, Ghironi and Méritz (2012): the inter-temporal elasticity of substitution is \( \rho = 1 \), the Frisch elasticity is \( \varphi = 4 \), the disutility of labour is normalized so that the steady state level of employment is equal to one and the elasticity of substitution across varieties is \( \theta = 3.8 \). The choice of \( \theta \) implies a markup as high as 35 percent. Most macroeconomic studies consider a higher \( \theta \) and a lower markup, in line with Rotemberg and Woodford (1999), who document a markup of about 18 percent in US data. I have checked that using \( \theta = 7.8 \) has no major consequences for the analysis.

The parameters of love for variety are probably the most difficult to motivate. I follow Bergin and Corsetti (2008) and set \( \gamma^M = 1 \) for consumption goods and \( \gamma^E = 1.2 \) for capital goods. I have also experimented with \( \gamma^M = \gamma^E \), so that the price of capital is constant in units of consumption. The effects on the performance of the model are negligible.

In the baseline calibration, entry costs are in terms of both wages and capital with weights equal to, respectively, \( \epsilon = 0.4 \) and \( 1 - \epsilon = 0.6 \). For ease of comparison with other studies, I consider calibrations with \( \epsilon = 0 \) (capital entry costs) and \( \epsilon = 1 \) (labor entry costs).

\(^8\) The shape parameter is such that \[ \sqrt{\frac{\kappa (S_{\text{min}})^2}{(\kappa-1)^2(\kappa-2)}} = 5.5. \]
The degree of nominal rigidity is $\alpha = 0.49$, the middle point from the interval of values estimated by Galí et al. (2001) for the US. This implies an average duration of nominal contracts of around a half year. The parameters of the Taylor rule draw on Bilbiie et al. (2008), $\phi_1 = 0.8$, $\phi_y = 0$ and $\phi_u = 0.3$. I have also considered positive values for the coefficient on output in the Taylor rule, in the range $\phi_y \in (0.4, 1.5)$, with no major consequences for the analysis. Finally, given the scope of the analysis, which focuses on matching key business cycle facts, I consider an annualized version of the productivity process in King and Rebelo (1999) as a useful benchmark. The correlation is $\rho_z = 0.815$ and the variance is $\sigma_z = 0.013$.

4.2 Impulse responses

This section provides a qualitative description of the mechanics of shock propagation at work in the model. For this purpose, Figure 1 and 2 depict the impulse responses of key endogenous variables to a one standard deviation shock to labor productivity. The vertical axis shows percentage deviations from the steady state (a value of, say, 1 denotes a 1 percent deviation) and the horizontal axis shows the number of periods (years) after the shock. The impulse responses refer to the baseline model, (solid black line), to a version with capital entry costs, $\epsilon = 0$, (dashed blue line), and a version with labor entry costs, $\epsilon = 1$, (dotted green line).
To start with, consider the baseline model. The rise in labor productivity makes the business environment more attractive for both incumbents and new firms. Incumbents take advantage of the productivity improvement by increasing their size (the intensive margin). The hike in firm size is high on impact, then it monotonically declines towards the steady state. Entrants start a new business (the extensive margin) whenever the equilibrium firm value, measured by the share price $\hat{v}_t$, covers entry costs. The response of entry is very large on impact and takes approximately seven years to return back to the steady state. The favourable business scenario reduces the incentives to leave the market, leading to a drop in firm exits. Entry of new firms together with less exits translate into a gradual increase in the number of producers over time. Notice that more product variety generates a negative welfare effect that is reflected in the drop in the relative price $\hat{r}_t$: the benefits of consuming more products is less than the benefit of consuming more of each product. For the same motive, it also generates a fall in the aggregate price level (not shown) and a fall in inflation through the Phillips curve. The drop in the product price increases demand per variety, boosting firms’ profits. On the other hand, it reduces profit margins so that the net effect on profits is in principle ambiguous. Interestingly, the baseline model can provide pro-cyclical profits together with counter-cyclical markups consistent with a large evidence. The capacity to capture these facts simultaneously constitutes a major achievement of entry models relative to standard (fixed variety) business cycle models.

The productivity rise leads to a less than proportional increase in consumption. This reflects the optimal allocation of resources between the manufacturing sector (consumption) and the start-up sector (entry). In cyclical upturns, the return on investments is high because the present share value is low relative to the future and because share payoffs (firms profits) are high. Agents are therefore

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9Studies that document counter-cyclical mark-ups in the US include, among others, Rotemberg and Woodford (1999) and Bils (1987). In contrast with these studies, Nekarda and Ramey (2013) find that mark-ups are pro-cyclical or a-cyclical in the US.
induced to postpone consumption in the future. In the model, the only way to transfer resources intertemporally is by creating a new firm. Finally, the increase in both investments and consumption lead hours and output above their steady state level throughout the transition. These effects are quite persistent.

Now, consider the variants with either capital or labour entry costs. The dynamics of most variables is qualitatively similar across entry cost specifications, except for that of profits, share prices and exit. As it will be clear soon, the capacity to reproduce a plausible behaviour of exit has important consequences for the overall performance of the model. With entry costs as wages there is a sharp fall in the number of exiting firms (up to 20 times as large as the shock) together with a hike in profits and firm value. The reason is a strong pro-cyclical behaviour of entry costs when these are measured as wages.

Mechanically, entry costs affect the exit rate through their impact on firm value (recall that in equilibrium firm value exactly covers entry costs). The value of the firm in turn is inversely related to the probability of exit. In cyclical upturns, the pressure on labour demand from incumbents and entrants raises wages and entry costs. High entry costs boost equity prices through the free entry condition in the model. This implies a strong incentive for incumbents to stay in the market and reduce exits. While a pro-cyclical behaviour of wages accords with evidence, however, the same cannot be said for entry costs. Despite the sparsity of studies, especially at the business cycle frequency, entry costs seem to be a-cyclical or at most negatively correlated with economic activity.

Entry costs move in a counter-cyclical way when they are measured by capital prices. The ability to replicate a more plausible behaviour for entry costs, however, is far from costless. Because of counter-cyclical profits, the model with capital entry costs provides an incentive for incumbents to leave the market. This is reflected in a positive response of exits at odds with the data.

4.3 Second moments

This section compares the second moments implied by the model with the evidence about business formation discussed above. Theoretical moments are averages across 500 simulations of the model for 2000 periods. As is common in most macroeconomic models with endogenous varieties, all variables

\footnote{Using the Doing Business Survey of the World Bank, Barseghyan and Di Cecio (2011) document that entry costs are negatively correlated with both output and the entry rate in a large panel of countries. On average, the correlation of start-up costs with real GDP per worker is -0.37 while the correlation with the entry rate is -0.15. The start-up cost is measured as a percentage of the economy’s income per capita. It includes all legal fees and professional and legal services when these are required by the law.}
that are measured in consumption units are divided by the real product price so as to net out the effect of changes in the range of available varieties. As stressed by Ghironi and Mélitz (2005), the correction is necessary because statistical measures of CPI inflation are unable to adjust for the availability of new products as in the welfare-based price index. In addition, theoretical variables are HP-filtered with a smoothing parameter of 6.25 consistent with the data.

In the data, entry and exit are a simple count of the number of firms that have entered or left the market in the previous year. The corresponding theoretical measures are, respectively, $N_t^e$ and $\delta t N_t$. Investments in the model are given by the real value of start-up investments $\nu t N_t^e / p_t^E$, a measure not directly comparable with investments in the data. Investments at the extensive margin in fact imply a different transmission mechanism compared to investments at the intensive margin. Nonetheless, the properties of these variables are similar: the creation of new firms contributes to the accumulation of a state variable, the stock of firms, which behaves much like capital in standard (fixed variety) business cycle models.

Table 2 reports summary statistics for consumption, investments, hours, entry, exit and the number of firms. The top rows display the standard deviation of each variable relative to the standard deviation of GDP while the bottom rows display the correlation of each variable with GDP. Columns refer to different parameterizations of the model except for the last one which reports US data from Table 1. Given the scope of the analysis, which is focused on the role of business formation for the propagation of shocks, I compare the baseline model with a version that features a given exit rate (fixed exit) and a version with a fixed number of firms (fixed variety). For ease of comparison with models that express entry costs in terms of wages or in terms of capital I provide simulations with, respectively, $\epsilon = 1$ and $\epsilon = 0$. 


The baseline model can reproduce important facts about firms dynamics. Entry and exit are far more volatile than output and the ranking of volatility is consistent with the data. Entry is pro-cyclical, exit is counter-cyclical, they are negatively correlated between each other (the correlation, not shown in Table, is -0.62) and both are correlated with the stock of firms (the correlation is, respectively, 0.11 and -0.16). Firm volatility is very close to the data. On a less positive tone, exits are too counter-cyclical and firms are too pro-cyclical compared to the data. Remarkably, the model can match these facts together with key properties of the US business cycle. In particular, consumption and investments are as smooth and pro-cyclical as the data. On the other hand, hours are too smooth and less pro-cyclical than the data.

A comparison across specifications, reveals interesting insights about the role of business formation. First, models that allow for some form of firm dynamics outperform the model with a fixed number of firms: the second moments of all variables are indeed closer to the data when firms are allowed to enter and/or exit the market. The fact that firm dynamics helps to improve the performance of artificial economies at matching the business cycle properties of the data was originally stressed by Bilbiie, Ghironi and Mélitz (2012) and confirmed by subsequent studies. My model makes no exception. Second, the baseline model fares favourably relative to the specification with a fixed exit rate. In particular, it generates a volatility of firms very close to the data. In addition,
it provides more plausible moments for investments and hours. The reason is counter-cyclical exit movements, which help to attenuate the excessive smoothness of macroeconomic variables that is typical of models with a given exit rate. I view the performance of the baseline model as a relative success. It generates second moments that are comparable with those of business cycle models with firm entry. In addition, it captures the properties of business formation in the data fairly well.

Consider now entry costs. I have experimented with the whole range of admissible values for $\varepsilon$ although only a few illustrative cases are reported in Table 2 for the sake of brevity. The model is sensitive to changes in the composition of entry costs, especially in the neighbourhood of extreme values. The baseline model, which features both labour and capital entry costs, performs unambiguously better than specifications with either labour ($\varepsilon = 1$) or capital costs ($\varepsilon = 0$). In particular, these specifications fail to generate a volatility of investments, entry, hours and firms as high as in the data. Moreover, the model with $\varepsilon = 0$ provides too volatile consumption and too smooth exits. In addition, it implies pro-cyclical exit at odds with the data. In the model with $\varepsilon = 1$, on the contrary, consumption is too smooth, exit is too volatile and too counter-cyclical.

A few studies have suggested the importance of entry/investment costs for the performance of theoretical economies at replicating key properties of the data. In my own previous work (Cavallari 2013a, 2013b), I show that measuring entry costs with product prices may help to capture important features of the international and the domestic business cycle, especially in models with sticky prices. This helps to replicate a plausible mechanism of monetary transmission, a point originally stressed by Bergin and Corsetti (2008). A monetary expansion, in fact, by reducing the price of capital in terms of consumption, fosters entry of new firms as in the data. With labour costs, on the contrary, the pressure on labour demand implied by the monetary easing raises wages and discourages entry. These studies are silent about the implications of entry costs for firm exit.

As discussed before, entry costs affect both business creation and destruction in my setup. The entry condition equalizes the value of the firm to the entry costs. The value of the firm in turn is negatively related to the probability of exit. Given all other conditions, an increase (a drop) in entry costs will reduce (increase) the incentives for firms to move in and out of the market. In the model, a low volatility of entry reflects a small incentive to adjust the start-up margin over the cycle, for example by creating a new business in cyclical upturns. In Table 2, the volatility is particularly low when entry costs are measured as wages, $\varepsilon = 1$. In order to see why, consider a rise in labour productivity. Marginal costs drop in the manufacturing and the start-up sector, similarly to what happens in a standard (fixed-variety) two-sector real business cycle model in the wake of an aggregate supply shock. The productivity improvement stimulates investments in production capacity (the intensive margin) as well as the creation of new firms (the extensive margin). Over
time, the pressure on labor demand from new and incumbent firms raises wages and entry costs. In contrast to the standard real business cycle model, here the hike in sunk costs discourages the creation of new firms relative to investments in the manufacturing sector. The shift out of the start-up sector reduces the volatility of entry and firms. On the other hand, the rise in entry costs boosts firm value through the free entry condition, giving a strong incentive to incumbent firms to stay in the market and reduce exits. The volatility of exits is therefore high.

The opposite occurs with capital entry costs, $\epsilon = 0$. Here, the productivity rise is akin to a shock in the manufacturing sector more than to an aggregate supply shock. Spillovers through factor demand are therefore much more moderate compared to the case with $\epsilon = 1$. Consequently, the volatility of entry is higher and that of exit is lower.

5 Conclusions

This paper studied the business cycle implications of product creation and destruction in a dynamic stochastic general equilibrium model with endogenous entry and exit. It compared the performance of the model at replicating important features of US data under alternative assumptions about firms’ dynamics and the composition of entry costs.

The contribution of the paper is twofold. First, it documents some new facts about business formation in the US. Firms’ entry and exit are far more volatile than output, they are negatively correlated between each other and both co-move with output. The stock of firms is less volatile than output and pro-cyclical. Then, it shows that my baseline model can replicate these facts fairly well. In addition, the model fares favourably relative to models with a fixed exit rate in capturing important properties of the US business cycle. In particular, it matches the smoothness of consumption and investments together with their correlation with output. The ability of the model to match the data is sensitive to changes in the composition of entry costs.

References


6 Appendix

6.1 The non-linear model

Consider a two-sector economy. The manufacturing sector, denoted by the superscript M, produces goods used for consumption and investment purposes. The start-up sector, denoted by the superscript E, creates new firms/varieties. There are \( N_t \) firms each producing a unique variety \( j \in (0, N_t) \) and \( L \) identical households. They consume a basket containing all the goods available in the market, supply labor to firms and own claims on firms’ profits.

The typical household supplies \( H_t \) hours of work each period for the nominal wage \( W_t \) and consumes a basket of goods \( C_t \). She maximizes utility over her entire lifetime \( E_t \left[ \sum_{s=t}^{\infty} \beta^{s-t} U(C_s, H_s) \right] \) at the subjective discount factor \( \beta \). The period utility is the additive-separable function \( U_t = \frac{(C_t)^{1-\rho}}{1-\rho} - \frac{\varphi x}{1+\varphi} (H_t)^{1+\varphi} \), where \( \rho > 0 \) is the inverse of the inter-temporal elasticity of substitution, \( \varphi \geq 0 \) is the Frisch elasticity of labor supply and \( \chi > 0 \) weights the disutility of labour.

The consumption basket takes the form \( C_t = A^M_t \left[ \int_0^{N_t} C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \), where \( A^M_t = N_t^\gamma \left[ \frac{\theta-1}{\theta} \right] \). As in Benassy (1996), the parameter \( \gamma^M \) measures consumers’ love for variety and the parameter \( \theta \geq 1 \) denotes the elasticity of substitution across varieties. The welfare-based price index is \( P_t = 1/A^M_t \left[ \int_0^{N_t} p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \).

Households enter each period with holdings of a nominal bond \( B_t \) and of a mutual fund share \( s_t \). The real value of a share in the mutual fund is \( v_t \). They receive labor income, interest income on bond holdings at the risk-free rate \( i_t \), dividend income on share holdings and the value of selling their initial share position. These resources are allocated between purchases of bonds and shares to be carried into next period and consumption. The period budget constraint (in units of consumption) is:

\[
\frac{B_t}{P_t} + s_t \left( N_t + N^E_t \right) v_t \leq \frac{B_{t-1}}{P_t} (1 + i_t) + s_{t-1} (1 - \delta_t) (v_t + d_t) \left( N_{t-1} + N^E_{t-1} \right) + \frac{W_t H_t}{P_t} - C_t \tag{11}
\]

where \( \delta_t \) is an endogenous rate of firm exit that will be defined soon.
Producers face a linear technology \( y_t(j) = Z_t H_t(j) + S_j \), where \( Z \) is a productivity shock and \( S_j \) is a firm-specific fixed factor. In each period, in addition to incumbent firms there is a finite mass of entrants, \( N_t^E \). As in Ghironi and Méliot (2005), entry implies a one period production lag: all firms entered in a given period are able to produce in all subsequent periods until they leave the market.

In departing from Ghironi and Méliot (2005), I assume that the probability of leaving the market \( \delta_t \) is a function of firm-specific scrap values \( S_j \). In particular, I assume that the scrap value of firms’ capital follows a Pareto distribution with shape \( \alpha \) and level \( S_{\text{min}} \). At the beginning of each period, before production and entry take place, firms decide to stay or leave the market by comparing the continuation and the exit value. They will leave whenever their market value, the equity price in the model, is below the scrap value draw. The probability of exit is therefore \( \delta_t = \left( \frac{\nu_t}{S_{\text{min}}} \right)^{-\alpha} \).

First-time entrants need to pay a sunk entry cost \( f_t^e \) to start production in period \( t + 1 \). The start-up of a new firm requires a combination of labor and capital \( N_t^E = (Y_t^E)^{1-\epsilon} (H_t)^{\epsilon} \) where \( \epsilon \in (0, 1) \) and capital consists of a composite basket of products \( Y_t^E = A_t^E \left[ \int_0^{N_t} y_t(j) \frac{\theta+1}{\theta} \right] \frac{\theta}{\theta-1} \).

Here, \( A_t^E = N_t^E \left[ (\frac{\theta}{\theta-1}) \right] \) is an indicator of efficiency of investment analogue to love of variety for consumption. The cost of entry is \( f_t^e = \left( P_t^E / P_t \right)^{1-\epsilon} (W_t / Z_t P_t)^{\epsilon} \) and the price of materials is \( P_t^E = 1/A_t^E \left[ \int_0^{N_t} p_t(j)^{1-\theta} \right] \frac{1}{\theta-1} \). Entrants start a new firm whenever its real value, \( v_t \), given by the present discounted value of the expected stream of profits \( \{d_s\}_{s=t+1}^\infty \), covers entry costs. The free entry condition is therefore \( v_t = f_t^e \).\(^{11}\) The timing of entry and exit together with the one-period production lag imply the law of motion for producers \( N_t = (1 - \delta_t) \left( N_{t-1} + N_t^e \right) \).

### 6.2 Equilibrium conditions

#### 6.2.1 Consumers

Consumers’ first order conditions are given by:

\[
\frac{(C_t)^{-\rho}}{P_t^e} = \beta E_t \left( \frac{(C_{t+1})^{-\rho}}{P_{t+1}^e} (1 + \delta_{t+1}) \right) \tag{12}
\]

\[
(C_t)^{-\rho} = \beta (1 - \delta_t) E_t \left[ \frac{d_{t+1} + \nu_{t+1}}{v_t} (C_{t+1})^{-\rho} \right] \tag{13}
\]

\[
C_t(j) = (A_t^M)^{\theta-1} \left( \frac{p_t(j)}{P_t} \right)^{-\theta} C_t \tag{14}
\]

\(^{11}\)The free entry condition holds as long as the mass of entrants in positive. In what follows, I assume that shocks are small enough for this condition to hold in every period.
\[
\frac{W_t}{P_t} = \chi (H_t)^{\frac{1}{\theta}} (C_t)^{\theta} \quad (15)
\]

### 6.2.2 Firms

Firms in the manufacturing sector face a downward-sloping market demand given by:

\[
y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} \left[ (A_t^M)^{\theta-1} C_t + (A_t^E)^{\theta-1} \left(\frac{P_t}{P_t^E}\right)^{-\theta} Y_t^E \right] \quad (16)
\]

I introduce nominal rigidities à la Calvo (1983). In each period a firm can set a new price with a fixed probability \(1 - \alpha\) which is the same for all firms, both incumbents and entrants, and is independent of the time elapsed since the last price change. In every period there will therefore be a share \(\alpha\) of firms whose prices are pre-determined.\(^{12}\)

Each firm sets the price for its own variety so as to maximize the present discounted value of future profits, taking into account market demand and the probability that she might not be able to change the price in the future. This yields:

\[
p_t(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} (\alpha \beta (1 - \delta_i))^k \frac{W_{t+k}}{Z_{t+k} P_{t+k} C_{t+k}^{\theta}} \frac{y_{t+k}(j)}{P_{t+k}^{\theta} C_{t+k}}}{E_t \sum_{k=0}^{\infty} (\alpha \beta (1 - \delta_i))^k \frac{y_{t+k}(j)}{P_{t+k}^{\theta} C_{t+k}}} \quad (17)
\]

Clearly, when \(\alpha = 0\) the optimal price implies a constant markup \(\frac{\theta}{\theta - 1}\) on marginal costs at all dates. With \(\alpha > 0\), prices are disconnected from marginal costs, implying time-varying profit margins. The expression above together with the definitions of \(P\) and \(P^E\), yield the Calvo state equations corrected for firm entry:

\[
(P_t)^{1-\theta} = (A_t^M)^{\theta-1} \left[ \frac{N_t}{N_{t-1}} (P_{t-1})^{1-\theta} + (1 - \alpha) N_t (p_t(j))^{1-\theta} \right] \quad (18)
\]

\[
(P_t^E)^{1-\theta} = (A_t^E)^{\theta-1} \left[ \frac{N_t}{N_{t-1}} (P_{t-1}^E)^{1-\theta} + (1 - \alpha) N_t (p_t(j))^{1-\theta} \right]
\]

In these equations, an increase in \(N_t\) may reduce or increase the aggregate price level depending on how much consumer value the benefit of spreading consumption over more products relative to the benefit of consuming more of each variety.

\(^{12}\)The pre-determined price is the average market price in the preceding period. The simplifying assumption that entrants behave like incumbents is without loss of generality. Allowing entrants to make their first price-setting decision in an optimal way would have only second order effects in my setup with Calvo pricing.
6.2.3 Aggregate constraints

Output of the manufacturing sector is defined as $Y_t^M \equiv \int_0^{N_t} p(j) y_t(j) dj$. The aggregation of market demand (16) across firms yields the market clearing condition $Y_t^M = C_t + p_t^E Y_t^E$. Aggregate accounting implies that GDP, i.e. output of the manufacturing sector plus output of the start-up sector, coincides with labor and profit income $Y_t = \frac{W_t}{P_t} H_t + d_t N_t$.

Labor market clearing requires labor supply to equalize labor demand from the manufacturing sector $H^M$ plus labor demand from the start-up sector $H^E$:

$$H_t \geq H_t^M + H_t^E = \int_0^{N_t} \frac{y_t(j)}{Z_t} dj + N_t^E \left[ \frac{\epsilon}{1 - \epsilon} \frac{p_t^E}{W_t} \right]^{(1 - \epsilon)}$$ (19)

The model is closed by specifying a monetary rule. I assume the monetary instrument is the one-period risk-free nominal interest rate, $i_t$, and monetary policy belongs to the class of feedback rules.

6.3 Steady state

The model is solved in log-deviation from a symmetric steady state equilibrium without inflation. Assuming $Z = 1$, the steady state of the economy is such that:

$$v = \delta^{-1} S \min, \quad N = \left( v \frac{\theta^e}{(1 - \theta)} \right)^{\frac{1}{\gamma - \gamma^E (1 - \epsilon)}}$$

Other variables are given by:

$$i = \frac{1 - \beta}{\beta}, \quad d = \frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} v, \quad \mu = \frac{\theta}{(\theta - 1)}, \quad \frac{p(j)}{P} = N_t^{\gamma - 1}, \quad \frac{p(j)}{P_t^E} = N_t^{\gamma^E - 1}$$

$$\frac{P^E}{P} = N_t^{\gamma - \gamma^E}, \quad w = \frac{N_t^{\gamma - 1} (\theta - 1)}{\theta}, \quad Y_t^M = d \theta N_t^\gamma, \quad H_t^M = d \theta, \quad Y = d N \left[ \theta N_t^{\gamma - 1} + \frac{\epsilon \beta \delta}{1 - \beta (1 - \delta)} \right]$$

$$C = v \left[ \frac{1 - \beta (1 - \delta) \theta N_t^\gamma - (1 - \epsilon) \beta \delta N}{\beta (1 - \delta)} \right], \quad H = \theta d \left[ 1 + \frac{\epsilon \beta \delta N_t^{2 - \gamma}}{1 - \beta (1 - \delta) (\theta - 1)} \right], \quad N_t^E = \frac{\delta}{(1 - \delta) N}$$

7 Derivation of inflation rates and markups (not for publication)

To derive the inflation rate (in log-linear terms), I linearize the optimal price (17) together with market demand (16) and the market clearing condition in the labor market (19), obtaining:
\[ E_t \sum_{k=0}^{\infty} \alpha_\beta (1-\delta_t)^k \left[ \hat{\omega}_{t,t+k} - \left( \rho + \frac{1}{\varphi} \right) \hat{C}_{t+k} + \left( 1 + \frac{1}{\varphi} \right) \hat{Z}_{t+k} - \frac{1}{\varphi} \hat{N}_{t+k} + \frac{\theta}{\varphi} \hat{\omega}_{t,t} \right] = 0 \]

Note that by definition \( \hat{\omega}_{t,t+k} = \hat{\omega}_{t,t} - \sum_{s=1}^{k} \pi_{t+s} \), which I substitute in the expression above. Using

\[ \hat{\omega}_{t,t} = (\theta - 1) A_t^M + \frac{\alpha}{1-\alpha} \pi_t + \frac{1}{(1-\alpha)(\theta-1)} \hat{N}_t - \frac{\alpha}{(1-\alpha)(\theta-1)} \hat{N}_{t-1} \]

and re-arranging gives the new-Keynesian Phillips curve corrected for firm entry:

\[ \pi_t = \zeta \left[ (\theta - 1) A_t^M + \left( \rho + \frac{1}{\varphi} \right) \hat{C}_t - \frac{1}{(1-\alpha)(\theta-1)} \hat{N}_t - \frac{(1+\varphi)}{\varphi} Z_t + \frac{\alpha}{(1-\alpha)(\theta-1)} \hat{N}_{t-1} \right] + \beta (1-\delta) E_t \pi_{t+1} \]

(20)

where \( \zeta = \frac{(1-\alpha)(1-\delta)(1-\alpha)}{\alpha(\varphi+\theta)} \). An analogous expression holds for inflation in the start-up sector, \( \pi_t^E \).

The expression for markups derives from a log-linear approximation to the definition \( \mu_t = \int_0^{N_t} \frac{p_t(j)Z_t}{W_t} dj \) where \( p_t(j) \) is given by equation (17).