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MODELLING ENTRY COSTS: DOES IT MATTER FOR BUSINESS CYCLE TRANSMISSION?

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Modelling entry costs: does it matter for business cycle transmission?

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Abstract

This paper studies the business cycle implications of entry costs in a dynamic stochastic general equilibrium model with firm entry and nominal rigidity. Simulations show that my baseline model matches the dynamics observed in the data fairly well. Remarkably, it overcomes the well-known difficulties of business cycle models in reproducing the persistence, smoothness and cyclicality of macroeconomic aggregates. I stress that capital entry costs are essential for these results.

Keywords: entry costs, firm entry, business cycle, investment costs.

JEL codes: E31; E32; E52

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1 Introduction

A novel line of research has stressed the role of endogenous firm entry and creation of new varieties in propagating business cycle fluctuations.[1] Some authors, as Bilbiee et al. (2007, 2012) and Ghironi and Mélitz (2005), focus on labor costs in the spirit of Grossman and Helpman (1991) and Romer (1990). In these models, start-up activities require labor inputs and entry costs coincide with labor marginal costs. Others, as Bergin and Corsetti (2008) and Cavallari (2012), assume investors need to buy materials (investment goods) for setting up a new firm, so that entry costs vary with their price. How to model entry costs is an open question. It has implications for aggregate accounting. Labor costs imply a wedge between output of the consumption sector and GDP that is absent in models with investment goods. More importantly, it may affect the mechanism of business cycle transmission. Investigating these issues is the main objective of this paper.

Entry costs are akin to investment costs in standard (fixed-variety) business cycle models. As for traditional models, there is some debate on the form of these costs. Specifically, the theoretical discussion is on the composition of investment/entry costs and the extent to which these are subject to nominal frictions. Regarding the former aspect, Cavallari (2012) shows that a tradable component in entry costs is essential for generating positive comovements in the international business cycle as those observed in the data. In this model, terms of trade movements spread the benefits of a productivity rise worldwide by creating new investment opportunities in high and low productivity economies. This weakens the incentive to move resources to the most productive economy at the root of the "comovement puzzle" in standard business cycle models.

Recent studies address the question of the role of nominal frictions in models with firm entry (see Bergin and Corsetti (2008), Lewis and Poilly (2012) and Uuksüla (2010), among others). Using US data on net business formation, they provide evidence of a negative relation between nominal interest rate innovations and investments in new firms (the so-called extensive margin).[2] Motivated by this evidence, Bilbiie et al. (2007) study a modification of their model with entry costs fixed in units of consumption. They note that fixed costs help to lead the theoretical responses of investments in line with the estimated ones although at the cost of implying pro-cyclical markups.

This paper contributes to the debate on entry costs along two dimensions. First, it provides a unified framework nesting capital and labor entry costs. Second it studies the implications of

[1] A non-exhaustive list of contributions includes: Bilbiee et al. (2007, 2012), Colciago and Etro (2010), Ghironi and Mélitz (2005), Bergin and Corsetti (2008), Jaymovich and Floetotto (2008), Lewis (2009), Cavallari (2007, 2010) and Russ (2007).

[2] Models with capital entry costs and sticky prices, as Bergin and Corsetti (2008), deliver theoretical responses in line with estimated responses. With labor entry costs, as in Uuksüla (2010), instead, limited participation models perform better than sticky price models in reproducing the response of investments observed in the data.

varying the composition of entry costs for the propagation of a broad range of business cycle shocks. Specifically, I will focus on productivity, government spending and monetary policy innovations.

As is common practice in endogenous entry models, producers are subject to a sunk entry cost, a one-period production lag and an exogenous exit shock. Each of them produces a unique variety in a monopolistic competitive market and sets the price of his product subject to nominal rigidity à la Calvo (1983). Financial markets are complete. In departing from previous contributions, I assume that the start-up of a new firm requires a combination of labor and capital, so that entry costs depend on the price of both investment goods and labor marginal costs. I will take an agnostic approach and investigate the performance of the model at replicating macroeconomic dynamics under alternative assumptions on the composition of entry costs.

Simulations show that my baseline model matches the dynamics observed in the data fairly well. Remarkably, it overcomes the well-known difficulties of business cycle models in capturing the persistence, smoothness and cyclicality of macroeconomic variables. I stress that the performance of the model is not robust to varying the composition of entry costs. In particular, allowing for a different composition of the investment and consumption baskets is essential for replicating the dynamics in the data. The intuition is that changes in the relative price of investment goods play an important role in the model, approximating the profit opportunities faced by potential investors over the cycle.

Comparing macroeconomic dynamics under various compositions of entry costs reveals interesting insights. Counter-cyclical entry costs, as those implied by a sticky-price model with investment goods, help to provide theoretical responses close to the estimated ones. Labor entry costs, on the contrary, have a pro-cyclical behavior that induces a positive response of new firms to interest rate and government spending innovations in contrast to what observed in the data. [3] In order to see why, consider an increase in the nominal interest rate that raises the real return on assets (bonds and shares). With labor entry costs, the rise in the return on shares requires a fall in today's price of equity relative to tomorrow's. The monetary tightening, in fact, restrains labor demand, thereby reducing real wages and entry costs. Consequently, more firms enter the market. The presence of capital entry costs works in the opposite direction. Insofar as a drop in aggregate demand is accompanied by a gradual decline in inflation, today's price of investment goods is above tomorrow's price, discouraging investments in new firms.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses

[3] Uusküla (2010) finds that a 1% increase in the Federal Funds rate rate leads to a 0.6% fall in the entry rate. Lewis (2009) shows that a rise in government spending has no significant impact on net business formation. In the model, the theoretical response of entrants is negative unless the spending shock is highly persistent. See also Totzek and Winkler (2010).

the solution strategy. Section 3 illustrates the performance of the model in reproducing the dynamics in the data. Section 4 contains conclusive remarks.

2 The economy

The economy is populated by a continuum of agents of unit mass indexed by i. Firms are monopolistic competitors, each producing a unique variety $j \in (0, N)$, where N is both the number of firms and the range of available varieties.

A typical agent supplies H_t hours of work each period for the nominal wage W_t and maximizes inter-temporal utility $E\left[\sum_{s=t}^{\infty} \beta^{s-t} U(C_s, H_s)\right]$, where C is consumption and β the subjective discount factor. The period utility is the additive-separable function $U_t = \frac{(C_t)^{1-\rho}}{1-\rho} - \frac{\varphi\chi}{1+\varphi}(H_t)^{\frac{1+\varphi}{\varphi}}$,with $\rho > 0$ and $\varphi \geq 0$. The consumption basket takes the form $C = \left[\int_0^N C(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{(\theta-1)}}$ with elasticity $\theta > 1$. The corresponding price index is $P = \left[\int_0^N P(j)^{1-\theta} dj\right]^{\frac{1}{(1-\theta)}}$.

Producers face an identical linear technology in the labor input $y_t(j) = Z_t H_t(j)$, where Z is an aggregate shock to labor productivity. In each period, in addition to incumbent firms there is a finite mass of entrants, N^e. As in Ghironi and Mélitz (2005), all firms entered in a given period are able to produce in all subsequent periods until they are hit by a death shock, which occurs with a constant probability $\delta \in (0, 1)$.

In order to start the production in period $t+1$, at time t an entrant needs to pay an exogenously given sunk entry cost f^e. In departing from previous contributions, I assume that the creation of a new firm requires a combination of labor and capital inputs $f^e N_t^e = (K_t)^{1-\epsilon}(Z_t H_t)^{\epsilon}$ where capital is given by a composite basket of investment goods $K = \left[\int_0^N K(j)^{\frac{\sigma-1}{\sigma}} dj\right]^{\frac{\sigma}{(\sigma-1)}}$ and $\epsilon \in (0, 1)$. Entry costs are therefore given by $f^e \left(P_t^K / P_t\right)^{1-\epsilon} (W_t / P_t Z_t)^{\epsilon}$ with $P^K = \left[\int_0^N P(j)^{1-\sigma} dj\right]^{\frac{1}{(1-\sigma)}}$. In this formulation, $\epsilon = 1$ nests the case of labor entry costs, examined by Bilbiie et al. (2012), Auray et al. (2012) and Cavallari (2007) among others. In a model with labor entry costs, entrants are required to hire f^e effective labor units at the cost W_t / Z_t. With $\epsilon = 0$, entrants are required to purchase f^e units of investment goods at a price P^K (see for instance Bergin and Corsetti (2008), Cavallari (2012) and Arespa (2012)). Without loss of generality, I follow these contributions and assume that capital required in the setup of new firms depreciates completely after one period. Note that the composition of the investment basket may differ from that of the consumption basket, namely $\theta \neq \sigma$. Clearly, when $\theta = \sigma$ and $\epsilon = 0$ entry costs are constant in real terms (i.e. in units of consumption), a case examined by Auray and Eyquem (2011) and Bilbiie et al. (2007). As will be clear soon, the composition of investment goods has relevant consequences for the dynamics of the model.

Entrants are forward looking and decide to start a new firm whenever its real value, υ, given by the present discounted value of the expected stream of profits $\{d_s\}_{s=t+1}^{\infty}$, covers entry costs. The free entry condition is given by:

$$\upsilon_t = E_t \left[\sum_{s=t+1}^{\infty} \beta (1-\delta) \left(\frac{C_{s+1}}{C_s} \right)^{-\rho} d_s \right] = f^e \left(P_t^K / P_t \right)^{1-\epsilon} \left(W_t / P_t Z_t \right)^{\epsilon} \tag{1}$$

The free entry condition holds as long as the mass of entrants in positive. Macroeconomic shocks are assumed to be small enough for this condition to hold in every period. Note that upon entry firms' profits are time-varying and can even turn negative for a while. This is a key difference relative to early models of frictionless entry, where the absence of sunk costs leads profits to zero in every period (see Rotemberg and Woodford (1991) and Blanchard and Giavazzi (2003), among others). The timing of entry and the one-period production lag imply the following law of motion for producers:

$$N_t = (1-\delta) \left(N_{t-1} + N_{t-1}^e \right) \tag{2}$$

I assume complete financial markets. Agents can invest their wealth in a set of nominal state-contingent government's bonds, B, that span all the states of nature Ω. In addition to bonds, they hold a share s of a well-diversified portfolio of firms. The budget constraint of a typical agent i is given by:

$$\sum_{\Omega} q_t(\Omega_{t+1}) \frac{B_{it}}{P_t} + s_t \left(N_t + N_t^e \right) \upsilon_t \leq \frac{B_{it-1}}{P_t} + s_{t-1} \left(\upsilon_t + d_t \right) + \frac{W_t H_{it}}{P_t} - C_{it} - T_{it} \tag{3}$$

where q is the bond price and T are lump sum taxes.

Finally, the government finances an exogenous stream of expenditures $\{G_t\}_{t=0}^{\infty}$ which has the same composition of the consumption basket by collecting taxes and issuing state-contingent debt. The period t government budget constraint is:

$$\sum_{i=0}^{1} T_{it} + \sum_{\Omega} q_t(\Omega_{t+1}) \frac{B_{t+1}}{P_t} = G_t + \frac{B_t}{P_t} \tag{4}$$

2.1 Equilibrium conditions

2.1.1 Consumers

Consumers' first order conditions are given by:

$$\frac{q_t(s_{t+1})}{P_t}(C_t)^{-\rho} = \beta E_t \left(\frac{C_{t+1}}{P_{t+1}}\right)^{-\rho} \tag{5}$$

$$(C_t)^{-\rho} = \beta(1-\delta) E_t \left[\frac{d_{t+1}+\upsilon_{t+1}}{\upsilon_t}(C_{t+1})^{-\rho}\right] \tag{6}$$

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} C_t \tag{7}$$

$$\frac{W_t}{P_t} = \chi (L_t)^{\frac{1}{\varphi}} (C_t)^{\rho} \tag{8}$$

2.1.2 Firms

Each producer sets the price for its own variety facing a downward-sloping market demand:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\theta} (C_t + G_t + (1-\epsilon)\upsilon_t N_t^e) \tag{9}$$

I introduce nominal rigidities à la Calvo (1983). In each period a firm can set a new price with a fixed probability $1-\alpha$ which is the same for all firms, both incumbents and entrants, and is independent of the time elapsed since the last price change. In every period there will therefore be a share α of firms whose prices are pre-determined.[4]

Each firm sets the price for its own variety so as to maximize the present discounted value of future profits, taking into account market demand and the probability that she might not be able to change the price in the future, yielding:

$$P_t(j) = \frac{\theta}{\theta-1} \frac{E_t \sum_{k=0}^{\infty} (\alpha\beta(1-\delta))^k \frac{W_{t+k}}{Z_{t+k}} \frac{y_{t+k}(j)}{P_{t+k} C_{t+k}^{-\rho}}}{E_t \sum_{k=0}^{\infty} (\alpha\beta(1-\delta))^k \frac{y_{t+k}(j)}{P_{t+k} C_{t+k}^{-\rho}}} \tag{10}$$

The above expression can be re-arranged in a more familiar form as:

$$P_t(j) = \frac{\theta}{\theta-1}(1-\alpha\beta(1-\delta))\frac{W_t}{Z_t} + \alpha\beta(1-\delta) E_t P_{t+1}(j) \tag{11}$$

Clearly, when $\alpha = 0$ optimal pricing implies a constant markup $\frac{\theta}{\theta-1}$ on marginal costs at all dates. With $\alpha > 0$, prices respond less than proportionally to a marginal cost shock, implying time-varying profit margins.

[4] The simplifying assumption that entrants behave like incumbent firms is without loss of generality. Allowing entrants to make their first price-setting decision in an optimal way would have only second order effects in my setup with Calvo pricing.

Aggregating (11) across firms and using the welfare-based consumer price index yields the Calvo state equation corrected for firm entry:

$$(P_t)^{1-\theta} = \alpha \frac{N_t}{N_{t-1}} (P_{t-1})^{1-\theta} + (1-\alpha) N_t (P_t(j))^{1-\theta} \tag{12}$$

Note that an increase in the number of producers over time reduces aggregate consumer prices and the more so the higher the elasticity θ.[5] This is a consequence of love for variety: a wider range of varieties raises the value of consumption per unit of expenditure, implying a fall in aggregate prices. An analogous state equation holds for the price of investment goods P^K.

2.1.3 Aggregate constraints

GDP is defined as $Y \equiv \int_0^N \frac{P(j)}{P} y(j) dj + \epsilon \upsilon_t N_t^e$ where the first addend is output of existing goods (used as consumption or investment goods) while the second is output devoted to the creation of new varieties.[6] As is apparent in the expression above, entry cost specification has relevant consequences for aggregate accounting. The presence of labor entry costs (i.e. $\epsilon \neq 0$) implies a wedge between output of the consumption sector and GDP that is absent in the model with only investment goods (i.e. $\epsilon = 0$). Goods market clearing requires output to equalize aggregate demand, i.e. $Y_t = C_t + G_t + N_t^e \upsilon_t$. Labor market clearing implies:

$$H_t \equiv \int_0^1 H_{it} di \geq \int_0^{N_t} \frac{y_t(j)}{Z_t} dj \ + \epsilon \frac{\upsilon_t N_t^e}{Z_t} \tag{13}$$

The model is closed by specifying a monetary policy rule. I assume the monetary instrument is the one-period risk-free nominal interest rate, i_t, and monetary policy belongs to the class of feedback rules.

2.2 The log-linearization

The model has no closed-form solution. It is log-linearized around a symmetric steady state with zero inflation (details in the Appendix). In the steady state, stochastic shocks are muted at all dates, $Z_t = G_t = 1$.

The Euler equation for bond holdings is given by:

[5] The point was originally made by Mélitz (2003).

[6] It is immediate to realize that GDP is the sum of labor and profit income, $Y_t = W_t H_t + d_t N_t$, consistently with the NIPA definition.

$$E_t\widehat{C}_{t+1} = \widehat{C}_t + \frac{1}{\rho}\left(\widehat{i}_t - E_t\pi_{t+1}\right) \tag{14}$$

where a hat over a variable denotes the log-deviation from the steady state, $\pi_{t+1} = \ln P_{t+1}/P_t$ is inflation and E is the expectation operator. In (14), an increase in the real interest rate raises the return on bonds, therefore making it more attractive to postpone consumption in the future.

The Euler equation for share holdings is:

$$E_t\widehat{C}_{t+1} = \widehat{C}_t + \widehat{v}_t + \frac{1}{\rho}E_t\left(\frac{i+\delta}{1+i}d_{t+1} + \frac{1-\delta}{1+i}\widehat{v}_{t+1}\right) \tag{15}$$

Arbitrage in financial markets equalizes the real returns on shares and bonds at all times.

Labor supply is given by:

$$\widehat{H}_t = -\rho\varphi\widehat{C}_t + \varphi\left(\widehat{W}_t - \widehat{P}_t\right) \tag{16}$$

Using the definition of GDP and the labor market equilibrium (13), it is convenient to derive an aggregate production function $\widehat{Y}_t = \widehat{H}_t + \widehat{Z}_t + \widehat{P}_{t,t}$ where $\widehat{P}_{t,t} \equiv \ln P_t(j)/P_t$ is the real price of each variety (in log-deviation).

Consider now the optimal price (10). Using market demand and (8), re-arranging and linearizing gives:

$$E_t\sum_{k=0}^{\infty}\alpha\beta\left(1-\delta\right)^k\left[\widehat{P}_{t,t+k} - \left(\rho+\frac{1}{\varphi}\right)\widehat{C}_{t+k} + \left(1+\frac{1}{\varphi}\right)\widehat{Z}_{t+k} - \frac{1}{\varphi}\widehat{N}_{t+k} + \frac{\theta}{\varphi}\widehat{P}_{t,t}\right] = 0$$

Note that by definition $\widehat{P}_{t,t+k} = \widehat{P}_{t,t} - \sum_{s=1}^{k}\pi_{t+s}$, namely changes in the relative price of a variety over time are given by the so-called variety effect, the first addend, less inflation. Using (12), the variety effect is:

$$\widehat{P}_{t,t} = \frac{\alpha}{1-\alpha}\pi_t + \frac{1}{(1-\alpha)(\theta-1)}\widehat{N}_t - \frac{\alpha}{(1-\alpha)(\theta-1)}\widehat{N}_{t-1}$$

With $\alpha = 0$, an increase in the number of producers raises the relative price of each variety and the more so the lower the elasticity of substitution θ. This effect is dampened with $\alpha > 0$. Combining the two equations above and re-arranging gives the new-Keynesian Phillips curve corrected for firm entry:

$$\begin{aligned}\pi_t &= \zeta\left[\left(\rho+\frac{1}{\varphi}\right)\widehat{C}_t - \frac{1}{(1-\alpha)(\theta-1)}\widehat{N}_t - \frac{(1+\varphi)}{\varphi}Z_t + \frac{\alpha}{(1-\alpha)(\theta-1)}\widehat{N}_{t-1}\right] \\ &\quad +\beta\left(1-\delta\right)E_t\pi^H_{t+1}\end{aligned} \tag{17}$$

where $\zeta = \frac{(1-\alpha\beta(1-\delta))(1-\alpha)}{\alpha(\varphi+\theta)}$. An analogous expression holds for inflation in the investment sector, $\pi_t^K = \ln P_t^K / P_{t-1}^K$, where σ replaces θ.

A log-linear approximation to the number of entrants is obtained from the aggregate resource constraint:

$$\widehat{N}_t^e = \frac{\theta\left(1-\beta\left(1-\delta\right)\right)}{\beta\delta}\widehat{Y}_t + \left(1 - \frac{\theta\left(1-\beta\left(1-\delta\right)\right)}{\beta\delta}\right)\left(\widehat{C}_t + \widehat{G}_t\right) - \upsilon_t \tag{18}$$

Note that there is a trade-off between investments in new varieties and consumption of existing goods (the coefficient on C and G is negative). The law of motion of firms is:

$$\widehat{N}_t = \left(1-\delta\right)\widehat{N}_{t-1} + \delta\widehat{N}_{t-1}^e \tag{19}$$

Optimal pricing (10) together with the definition of aggregate markup $\mu \equiv \int_0^N \frac{P(j)}{W/Z}dj$ yield a useful expression for markups:

$$\widehat{\mu}_t = \alpha\beta\left(1-\delta\right)\left(\widehat{P}_t - \widehat{P}_{t,t-1} + \pi_t\right) \tag{20}$$

Markups rise above the steady state level so long as variety prices grow more rapidly than inflation.

As will be apparent soon, markup movements play a key role in the model. To begin with, they affect the expected dividends from investing in a new firm, influencing entry behavior through the free entry condition (1). A change in the stock of producers, in turn, modifies the allocation of resources between production of existing goods and creation of new varieties. The dynamics of these effects crucially depends on entry cost specification. In a model where entry requires only capital inputs (i.e., with $\epsilon = 0$) employment is entirely devoted to the production of existing goods while the creation of new varieties entails a one-period production lag. The allocation of resources between production of existing goods and creation of new varieties is therefore predetermined in each period. This is clearly not true when labor inputs are used in the production of new varieties.

With $\epsilon \neq 0$, the model is isomorphic to a two-sector economy where one sector produces existing goods and the other sector creates new varieties. The labor market clearing condition (13) then determines the allocation of resources between these two sectors at each time. In log-deviations this yields:

$$\left(\widehat{H}_t + \widehat{Z}_t\right)\left(1+\tau\right) = \widehat{Y}_t - \widehat{P}_{t,t} + \tau\widehat{N}_t^e$$

where $\tau \equiv \frac{\beta\delta}{(\sigma-1)(1-(1-\delta)\beta)}$.

The condition above is redundant when $\epsilon = 0$. In order to see why, consider aggregate markups. It is immediate to verify that they coincide with the inverse of the labor share, $\frac{YP}{WH}$, so long as $\epsilon = 0$,

implying an inverse relation with real wages $\widehat{W}_t - \widehat{P}_t = \widehat{Z}_t - \widehat{\mu}_t + \widehat{P}_{t,t}$. Aggregate hours can therefore be determined by substituting real wages into labor supply (8).

The log-model is closed by specifying a monetary policy rule. In what follows, I will consider a Taylor rule $\widehat{i}_t = \phi_i \widehat{i}_{t-1} + \phi_\pi \pi_t + \phi_y \widehat{Y}_t$ with interest rate smoothing (Taylor (1983)). These rules are empirically plausible, especially in the last few decades when the objective of price stability has gained a major role in monetary policy-making. Interest smoothing accounts for the need to reduce swings in interest rates in an environment characterized by long and variable lags in monetary transmission.

3 Simulations

The model is simulated using first-order perturbation methods. Business cycle volatility is driven by productivity and government spending shocks.

3.1 Calibration

The model is calibrated to the United States. In the simulations, periods are interpreted as quarters and $\beta = 0.99$ as is usual in quarterly models of the business cycle. The size of the exogenous exit shock is set at $\delta = 0.025$ as in Bilbiie et al. (2007) to match a 10 percent rate of job destruction per year as found in the US.

The elasticity of substitution among consumption goods is $\theta = 7.88$ as in Rotemberg and Woodford (1999), yielding an average markup of approximately 18 percent as in US data. The elasticity of substitution among investment goods is set at $\sigma = 10.83$ to capture the elasticity found in the US investment sector compared to that in the consumption sector.[7] Studies based on micro data usually find a much lower θ, roughly around 4, while there is no comparable evidence, to the best of my knowledge, for the investments sector. I have experimented with $\theta = 4$ and $\sigma = 5.5$ so as to maintain the relative size of these elasticities unchanged, obtaining qualitatively identical results. [8]

In the baseline calibration the share of labor and capital in entry costs are equal to, respectively, $\epsilon = 0.4$ and $1 - \epsilon = 0.6$ so as to capture a high capital intensity of setup costs. For ease of comparison with other contributions in the literature, I will also consider the cases of $\epsilon = 0$ (capital entry costs) and $\epsilon = 1$ (labor entry costs). Other preference parameters are $\varphi = 2.13$ as in Rotemberg and

[7] Using BEA input-output tables, Floetotto et al. (2009) estimate the elasticities in US investment and consumption sectors. They find that investment goods are more substitutable than consumption goods by a factor of roughly 0.72 percent.

[8] These results are available upon request.

Woodford (1999) and $\rho = 1$ as in Bilbiie et al. (2007).

Galì et al. (2001) estimate a value for the degree of nominal rigidity between 0.407 and 0.66 in the US. I take the middle point from this interval and set $\alpha = 0.49$, implying an average duration of nominal contracts of 2.3 quarters.

The exogenous processes for productivity and government spending follow each an AR(1) process in logs: $Z_t = \rho_z Z_{t-1} + \epsilon_t^z$ and $G_t = \rho_g G_{t-1} + \epsilon_t^g$ where the innovations ϵ_t^z and ϵ_t^g are distributed normally and independently of each other with variance σ_z^2 and σ_g^2 respectively. I set $\rho_z = 0.975$ and $\sigma_z^2 = 0.0072$ as in King and Rebelo (1999). The parameters for government spending are set to $\rho_g = 0.97$ and $\sigma_g^2 = 0.027$ as in Chari and Kehoe (1999). The parameters of the Taylor rule draw on Bilbiee et al. (2007), $\phi_i = 0.8$, $\phi_y = 0$ and $\phi_\pi = 0.3$. I have also considered positive values for the coefficient on output in the Taylor rule, in the range $\phi_y \in (0.4, 1.5)$, without remarkable changes in qualitative results. Finally, as fixed costs do not affect the dynamics of the model I set $f^e = 1$ without loss of generality.

3.2 Moments

To evaluate the properties of the model, this section computes the second moments of key macroeconomic variables and compares them to those of the data and those implied by the baseline model in Bilbiie et al. (2012), hereafter BGM. This latter provides a natural benchmark as a workhorse model in business cycle studies with firm entry.

In comparing the model to properties of the data, theoretical variables are divided by the relative price $P(j)_t/P_t$ so as to net out the effect of changes in the range of available varieties (for any variable X the corrected measure will be $X_t^R = P_t X_t / P(j)_t$). As stressed by Ghironi and Mélitz (2005), the correction is necessary because statistical measures of CPI inflation are unable to adjust for availability of new products as in the welfare-based price index. In the model, investments are measured by the real value of household investments in new firms ($\upsilon^R N^e$).

Table 1 reports statistics of the model's artificial time series together with statistics in US data and in the BGM model. As with the data, statistics refer to Hodrey-Prescott filtered variables with smoothing parameter of 1600. The reported statistics are averages across 200 simulations. The first column displays the moments implied by my baseline model ($\epsilon = 0.4$) , the second column refers to a variant with labor entry costs ($\epsilon = 1$), the third column reports the moments generated by the BGM model and the last column reports US data from Table 1 in Rotemberg and Woodford (1999).

Table 1:

	$\epsilon = 0.4$			$\epsilon = 1$			BGM model			US data		
	σ_X/σ_Y	σ_{XY}	ρ_X	σ_X/σ_Y	σ_{XY}	ρ_X	σ_X/σ_Y	σ_{XY}	ρ_X	σ_X/σ_Y	σ_{XY}	ρ_X
C^R	0.78	0.89	0.97	0.87	0.90	0.97	0.44	0.95	0.75	0.75	0.76	0.78
H^R	0.87	0.92	0.94	0.45	0.56	0.97	0.62	0.98	0.67	1.10	0.88	0.90
$\upsilon^R N^e$	3.08	0.81	0.91	1.12	0.99	0.97	4.18	0.99	0.67	2.99	0.79	0.87
μ^R	0.45	-0.65	0.89	0.21	-0.89	0.94	-	-	-	0.60	-0.28	0.79

σ_X is the standard deviation of variable X, σ_{XY} is the correlation of variable X with output, Y, and ρ_X is the auto-correlation of variable X.

The baseline model matches the dynamics observed in the data fairly well.[9] Remarkably, it overcomes the well-known difficulties of standard business cycle models in capturing the persistence, smoothness and output correlation of macroeconomic variables. In this respect it fares better than the BGM model as well. Additionally, my model replicates the synchronization of output and markups. Markups, however, are far more counter-cyclical than in the data.[10]

In my model, markup dynamics is due to nominal frictions (markups are constant with flexible prices). This is by no means essential for replicating counter-cyclical markups. Early studies, such as Jaimovich and Floetotto (2008), Colciago and Etro (2010) and Bilbiie et al. (2012) have shown this stylized fact without relying on the sticky price assumption. In these contributions, however, markup volatility is in general underestimated. Colciago and Etro (2010) stress the need for further work on the microfoundations of market structure in order to match the volatility of profits and markups better. I stress that nominal frictions also play an important role in this regard.

Comparing theoretical moments in the two scenarios considered provides interesting insights on the role of entry costs. First and foremost, overlooking capital costs dramatically deteriorates the performance of the model. With $\epsilon = 1$, the volatility of investments drops to a value as low as 1.10. In the model, a low volatility reflects a weak incentive to setup a new firm as a consequence of endogenous movements in entry costs. In order to see why, consider an unexpected rise in productivity. Labor costs drop on impact, thereby attracting new entrants. Over time, an increasing number of producers pushes on labor demand, raising wages and discouraging entry. Investment volatility therefore falls. This is in contrast to the excessive investment volatility implied by standard real business cycle models as well as by endogenous entry models à la Bilbiee et al. (2012). In these settings, nominal frictions are either overlooked or they are represented by costs of price adjustments. The deterrent effect of endogenous movements in labor costs is either absent (in flexible price models)

[9] Entry behaves similarly to investments (Lewis, 2009). In US data, the correlation between output and net entry measured as Net Business Formation is 0.71. The standard deviation of NBF relative to that of output is 2.19.

[10] In the baseline BGM model markups are constant. They are counter-cyclical in the variant with translog preferences.

or it can be mitigated by offsetting changes in markups or in other sources of marginal costs. With costs of price adjustment, for instance (as in Bilbiee et al.(2007), Auray and Eyquem (2011) and Auray et al. (2012)), the pressure on wages implied by entry is typically accommodated by a decline in firms' markups.

Second, note that the baseline model implies movements in the price of investment goods, P^K/P, that are absent with $\epsilon = 1$ and this may help to understand the reasons why the performance of the model is so different in the two cases. In order to address the issue in more detail, I have experimented with entry costs fixed in units of consumption (i.e., $\epsilon = 0$ and $\theta = \sigma$) so that $P^K/P = 1$. [11] The performance of the model deteriorates displaying excessive volatility for consumption (equal to 0.89) and too low volatilities for hours and markups (equal to 0.28 and 0.22, respectively). The reason is easy to grasp. With fixed entry costs, a rise in aggregate productivity implies a higher productivity in the sector that produces existing goods. Agents have therefore a strong incentive to move resources towards current production, by smoothing labor effort over time.[12] Differences in the composition of the consumption and investment baskets are therefore essential for mitigating this incentive and reproducing the dynamics of investments observed in the data.

3.3 Impulse responses

Up to now, I have shown that the performance of the model may not be robust to varying the composition of entry costs. In this section, I will investigate the mechanism of business cycle transmission in more detail so as to provide further insights on the role of entry costs. For ease of comparison with other contributions in the literature, I will focus on the polar cases of only labor and only capital entry costs.

Figure 1 shows impulse response functions of selected macroeconomic variables for a one-standard-deviation shock to productivity. The vertical axis shows percentage deviations from the steady state (a value of, say, 0.01 denotes a 1 percent deviation) and the horizontal axis shows the number of periods after the shock. For consistency with second moments, the shock has a persistence of 0.975. The impulse responses are displayed in a variant of the baseline model with investment goods, $\epsilon = 0$, (solid line) and in a variant with labor entry costs, $\epsilon = 1$, (dashed line).

A rise in productivity makes the business environment more attractive for new firms. The reason is an increase in the real return on assets (bonds and shares) brought about by a decrease in today's

[11] This case is examined by Bilbiie et al. (2007) and Auray and Eyquem (2011) among others.

[12] As well known, a strong incentive to smooth labor effort is at work in a flexible price economy. In my setup, this is apparent considering equation (11) with $\alpha = 0$. With flexible prices, nominal wages and prices move in unison, implying that real wages and hours worked are fairly stable over the cycle.

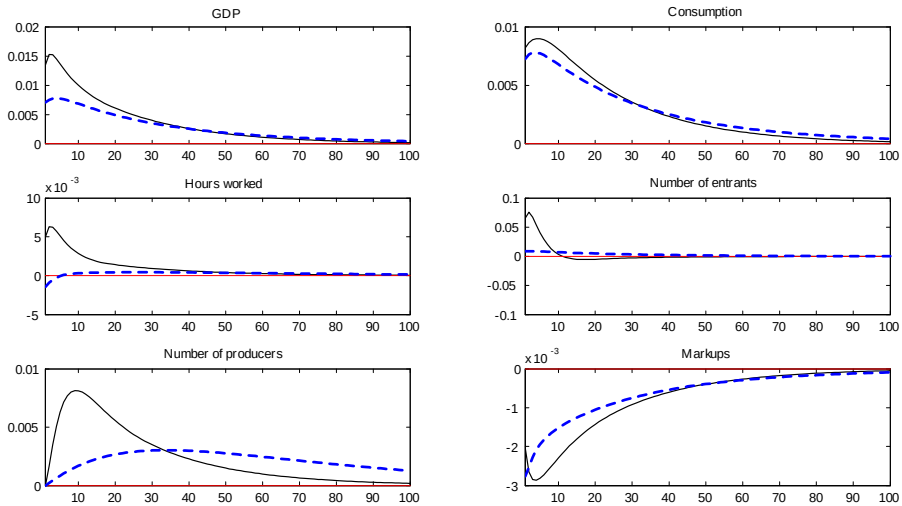

Figure 1: IRF to a 1% increase in productivity in the model with capital entry costs (solid line) and with labor entry costs (dashed line)

price of equity ν_t (the value of the firm) relative to tomorrow's $\widehat{v}_{t+1}$. Higher investments in turn translate into a gradual increase in the number of producers over time. Consequently, consumption and GDP raise above the steady state. Note that markups are counter-cyclical as observed in the data.[13] In my setup, the behavior of markups is a consequence of firm entry moving aggregate prices and marginal costs in opposite directions. On the one side, an increase in the number of producers, by pushing on labor and investment demand, raises marginal costs. On the other side, it reduces aggregate prices through the variety effect.

Comparing the responses in the two scenarios considered reveals notable differences. First, investments in new firms exhibit a hump-shaped pattern when entry requires capital inputs (similar to what observed in the data) that is absent with labor entry costs. The hump reflects opposing forces at work over the cycle. The productivity rise reduces the price of investment goods in the first part of the transition, i.e. P^K falls short of P. Over time, as the real return on assets diminishes, this relation inverts and investments in new firms become negative for a while before converging to the steady state. With labor entry costs, instead, the value of the firm $\widehat{v}_t$ declines in a gradual way. Labor marginal costs fall in proportion to the productivity rise on impact and then monotonically increase under the pressure of labor demand.

Secondly, investments in new firms are much less volatile with labor entry costs, in line with

[13] Studies based on different methodological approaches converge on the view that markups are counter-cyclical in major economies. In the US, this is indeed the case for studies using mostly aggregate data as Rotemberg and Woodford (1999) as well as two digit industry level data as in Bils (1987).

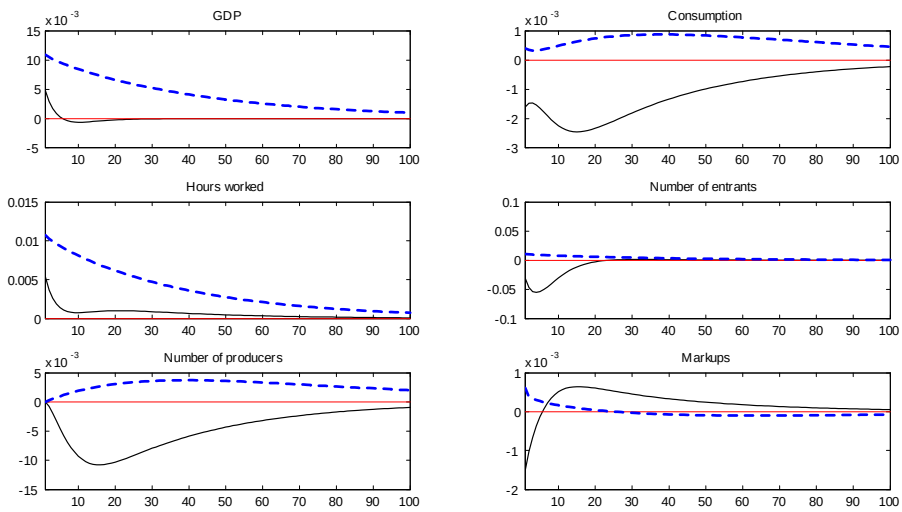

Figure 2: IRF to a 1% increase in government spending in the model with capital entry costs (solid line) and with labor entry costs (dashed line)

the theoretical moments discussed above. As before, the reason is a strong deterrent effect of pro-cyclical movements in real wages. Last but not least, a moderate increase in the number of producers implies a relatively stable path for hours worked in contrast to the sharp response in the model with investment goods.

Next consider an aggregate demand shock represented by a 1 standard deviation increase in government spending financed by lump-sum taxation (see Figure 2). For consistency with second moments, the persistence of the public spending shock is 0.97. The fiscal expansion induces agents to work more in the current period in anticipation of a future increase in taxes (negative inter-temporal wealth effect). The expansion of aggregate demand combined with an increase in hours worked lead GDP above the steady state, far more sharply with labor entry costs than in the model with investment goods. A casual inspection of Figure 2 reveals that the overall government spending multiplier in the terminology of Uhlig (2010) is far below unity in the model with investment goods. This is consistent with evidence of a very low value for the multiplier of government spending in the United States in line with a general tendency among OECD economies.[14] At odds with this evidence, the multiplier is above unity in the variant with labor entry costs.

A low fiscal multiplier is a consequence of crowding out of private expenditure. In the model with investment goods, both consumption and investments decline. In order to see why, consider

[14] Mountford and Uhlig (2009) estimate a value for the public spending multiplier in the US as low as 0.65 on impact and -1 in the long run. For recent evidence on fiscal multipliers in OECD economies see for instance Corsetti, Gernot and Mueller (2012) and Ilzetzky et al. (2012))

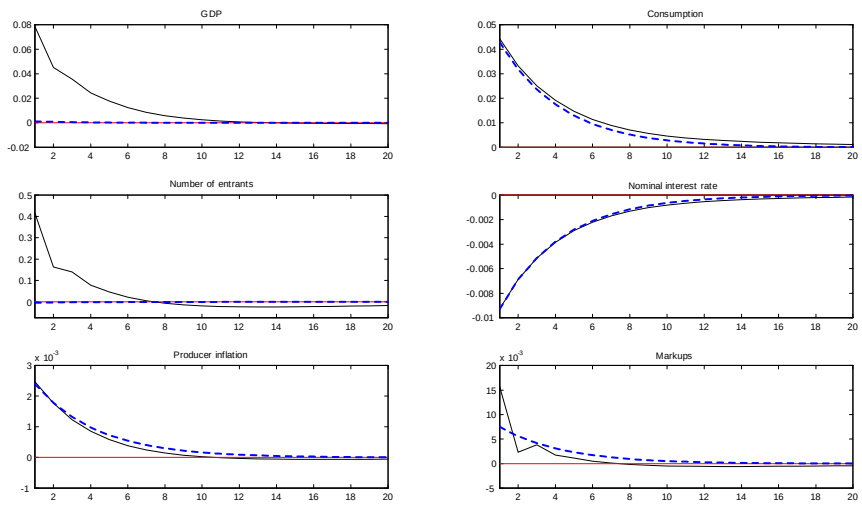

Figure 3: IRF to a 1% temporary interest rate cut in the model with capital entry costs (solid line) and in the variant with labor entry costs (dashed line)

the free entry condition in the model (1). Since the number of producers is fixed initially, the rise in aggregate demand has a positive effect on firms' profits, d_t. However, this may not be sufficient for inducing entry. A potential investor will decide to enter the market only insofar as these profit opportunities are expected to be persistent enough, i.e. if $d_{t+1} + \widehat{v}_{t+1}$ is larger than today's price of equity $\widehat{v}_t$. A low persistence of the government spending shock combined with a decline in tomorrow's price of equity work in the direction of reducing these expectations below $\widehat{v}_t$, hence discouraging entry. Consequently, the number of producers falls below the steady state level. This, in turn, curbs dividend incomes (i.e., $d_t N_t$ falls), thereby contributing to reduce wealth and private consumption.

With labor entry costs, on the contrary, private consumption and investments are positive (although small) throughout the transition. The reason is a decline in today's real wages relative to tomorrow's brought about by a slow adjustment of prices in the face of an expanding aggregate demand. Potential investors therefore anticipate persistent profit opportunities from setting up a new firm (and the more so the slower the adjustment of prices) and find it convenient to enter the market at an early stage of the transition. The consequent rise in the number of producers, in turn, contributes to increase consumers' wealth and to boost consumption.

The findings above suggest that the composition of entry costs and particularly the extent to which these costs are subject to nominal rigidity may alter the mechanism of business cycle transmission. It is therefore worth illustrating the monetary transmission at work in the model in more detail. For this purpose, I consider a purely temporary 1 standard deviation drop in the nominal interest rate (see Figure 3).

The monetary easing boosts aggregate demand as long as prices are sticky, leading to a spike in consumption. Over time, as prices slowly return to their natural levels, consumption converges to the steady state. The dynamics of consumption, inflation and markups is almost identical in the two specifications considered. The rise in consumption reflects a drop in the real interest rate, i.e. a drop in the return on bonds. Arbitrage in financial markets requires the real return on shares to fall as well. The decrease in the real return on shares is brought about by a fall in the return $(\widehat{v}_{t+1} + d_{t+1})$ relative to today's price of equity $\widehat{v}_t$. How this is accomplished crucially depends on entry costs specification.

The price of equity is tied to the cost of acquiring investments goods by the free entry condition (1) in the model. In the variant with investment goods, this cost is $\widehat{v}_t = \widehat{P}_t^K - \widehat{P}_t$. As goods prices adjust only gradually with nominal frictions, today's cost of equity falls relative to tomorrow's, favoring investments in new firms. This effect is stronger the higher the elasticity of substitution in the investment sector σ relative to that in the consumption sector θ. In the model with labor entry costs, instead, the price of equity is tied to labor marginal costs, i.e. $\widehat{v}_t = \widehat{W}_t - \widehat{P}_t - \widehat{Z}_t$. Under the pressure of labor demand, these costs rise and eventually offset the positive impact of inflation. As a consequence, investments in new firms slightly decline on impact.

4 Conclusions

This paper studied the implications of entry costs for the propagation of business cycle fluctuations. Building on a dynamic stochastic general equilibrium model with firm entry and nominal rigidity, it compared the performance of the model in replicating the dynamics in the data under various assumptions on the composition of these costs.

Simulations show that my baseline model matches the dynamics observed in the data fairly well. Remarkably, it overcomes the well-known difficulties of business cycle models in reproducing the persistence, smoothness and output correlation of macroeconomic aggregates. I stress that the performance of the model is in general not robust to varying the composition of entry costs. Specifically, allowing for time-varying capital entry costs turns essential for reproducing the dynamics in the data.

Furthermore, capital entry costs help to replicate responses to productivity, interest rate and government spending innovations close to those estimated. The performance of the model deteriorates also along this dimension when entry costs exclusively require labor inputs.

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5 Appendix

5.1 Steady state

The model is solved in log-deviation from a symmetric steady state equilibrium without inflation. Assuming $Z = G = 1$, the steady state of the economy is such that:

$$N = \left(\frac{\theta\left(1-\beta\left(1-\delta\right)\right)-\delta\beta}{\beta\left(1-\delta\right)} \right)^{\frac{\theta-1}{2-\theta}}$$

Other variables are given by:

$$\begin{aligned} i &= \frac{1-\beta}{\beta}, \quad v=1, \quad d=\frac{\left(1-\beta\left(1-\delta\right)\right)}{\beta\left(1-\delta\right)}, \quad \mu=\frac{\theta}{\left(\theta-1\right)}, \quad \frac{P(j)}{P}=N^{\frac{1}{\theta-1}} \\ C &= \theta N\left[\frac{1-\beta\left(1-\delta\right)}{\beta\left(1-\delta\right)}-\frac{\delta}{\theta\left(1-\delta\right)}\right], \quad L=\theta d N^{\frac{2-\theta}{1-\theta}}, \quad Y=\theta d N, \quad N^{e}=\frac{\delta}{\left(1-\delta\right)}N \end{aligned}$$

5.2 Loglinear model

Loglinearized conditions for households are:

$$
\begin{aligned}
E_t\widehat{C}_{t+1} &= \widehat{C}_t + \frac{1}{\rho}\left(\widehat{i}_t - E_t\pi_{t+1}\right) \\
E_t\widehat{C}_{t+1} &= \widehat{C}_t + \widehat{\upsilon}_t + \frac{1}{\rho}E_t\left(\frac{i+\delta}{1+i}d_{t+1} + \frac{1-\delta}{1+i}\widehat{\upsilon}_{t+1}\right) \\
\widehat{H}_t &= -\rho\varphi\widehat{C}_t + \varphi(\ \widehat{W}_t - \ \widehat{P}_t)
\end{aligned}
$$

Loglinearized conditions for firms are:

$$
\begin{aligned}
\widehat{N}_t &= (1-\delta)\,\widehat{N}_{t-1} + \delta\widehat{N^e}_{t-1} \\
\widehat{\mu}_t &= \alpha\beta\,(1-\delta)\left(\widehat{P}_{t,t+1} - \widehat{P}_{t,t} + E_t\pi_{t+1}\right) \\
\pi_t &= \zeta MC_t + \beta\,(1-\delta)\,E_t\pi_{t+1}
\end{aligned}
$$

where MC denotes an index of current marginal costs defined by the term in squared brackets in equation (17) in the main text.

Other log-linear equilibrium conditions are:

$$
\begin{aligned}
\widehat{P}_{t,t} &= \frac{\alpha}{1-\alpha}\pi_t + \frac{1}{(1-\alpha)(\theta-1)}\widehat{N}_t - \frac{\alpha}{(1-\alpha)(\theta-1)}\widehat{N}_{t-1} \\
\widehat{P}^K_{t,t} &= \frac{\alpha}{1-\alpha}\pi^K_t + \frac{1}{(1-\alpha)(\sigma-1)}\widehat{N}_t - \frac{\alpha}{(1-\alpha)(\sigma-1)}\widehat{N}_{t-1} \\
\widehat{Y}^J_t &= \widehat{Z}^J_t + \widehat{H}^J_t + \widehat{P}^J_{t,t} \\
\widehat{N}^e_t &= \frac{\theta\,(1-\beta\,(1-\delta))}{\beta\delta}\widehat{Y}_t + \left(1 - \frac{\theta\,(1-\beta\,(1-\delta))}{\beta\delta}\right)\widehat{C}_t - \widehat{\upsilon}_t \\
\widehat{W}_t - \widehat{P}_t &= \widehat{Z}_t - \widehat{\mu}_t + \widehat{P}_{t,t} \quad \text{with } \epsilon = 0 \\
\left(\widehat{H}_t + \widehat{Z}_t\right)(1+\tau) &= \widehat{Y}_t - \widehat{P}_{t,t} + \tau\widehat{N}^e_t \quad \text{with } \epsilon \neq 0 \\
\widehat{\upsilon}^J_t &= (1-\epsilon)\left(\widehat{P}^K_{t,t} - \widehat{P}_{t,t}\ \right) + \epsilon\left(\widehat{W}_t - \widehat{P}_t - \widehat{Z}_t\right)
\end{aligned}
$$

The model is closed with the interest rate rule indicated in the text.