Firms’ entry, monetary policy and the international business cycle

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Abstract

This paper studies the role of producer entry for global monetary policy and the propagation of the international business cycle in a two-country general equilibrium (DSGE) model with monopoly distortions and imperfect price adjustment. It introduces endogenous entry into the model of Benigno and Benigno (2008) so as to provide a first step towards the joint determination of exchange rates and product varieties under interest rate rules. The paper shows that endogenous product variety has relevant consequences for the dynamics of the exchange rate and the terms of trade, highlighting a novel channel of international cyclical transmission. The paper provides numerical examples of responses to country-specific shocks to aggregate productivity, entry costs and monetary policy under plausible interest rate settings with both floating and fixed exchange rates and compare them to their flexible price counterpart.

Keywords: product variety, firm entry, international business cycle, monetary policy, interest rate rules, exchange rate regimes.

JEL codes: E31; E32; E52

1 Introduction

This paper studies the role of producer entry for global monetary policy and the propagation of the international business cycle in a two-country general equilibrium (DSGE) model with monopoly distortions and imperfect price adjustment. The paper belongs to a recent line of research stressing the

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implications of producer entry and product creation for the business cycle propagation and policy.\textsuperscript{1} From a theoretical perspective, these models significantly improve upon their fixed-variety predecessors where monopolistic competition has the unappealing consequence of implying permanent positive profits. The introduction of more realistic assumptions on firms entry, in turn, ameliorates the capacity of artificial economies to reproduce many empirical patterns of macroeconomic aggregates over the cycle. Bilbiie, Ghironi and Mélitz (2007) show that the second moments generated by their model with endogenous entry and nominal rigidity come very close to the second moments in the data, outperforming the typical fixed-entry real business cycle model as well as their own model with endogenous entry and flexible prices (Bilbiie, Ghironi and Mélitz (2005)). Furthermore, these models can explain a number of empirical regularities that are at odds with the assumption of fixed variety, as the pro-cyclicality of entry and profits and the counter-cyclicality of markups. A growing number of contributions is therefore devoted to studying the consequences of endogenous product variety for monetary policy. A general finding is that sticky prices can distort entry behavior in a number of ways, thereby creating a new motive for cyclical stabilization and a new role for monetary policy in welfare maximization. It is argued that well-designed monetary rules can help replicate the business cycle patterns that would prevail with flexible prices. To this end, producer price stability typically emerges as a desirable policy prescription.\textsuperscript{2}

Most contributions in the field refer to closed economies and focus on internal dimensions of monetary policy. Russ (2007) and Cavallari (2007, 2009) provide notable exceptions by developing open-economy models with endogenous entry and sticky prices. Russ analyzes the implications of exchange rate variability for foreign direct investments. She finds that monetary volatility may have a positive or a negative impact on foreign investments depending on whether it originates at home or abroad. In my own previous work, I focus on the choice between trade and foreign investments. Cavallari (2007) shows that investments in domestic markets and abroad can be sub-optimally low whenever local monetary stabilization is incomplete. Cavallari (2009) finds that various dimensions of monetary uncertainty can affect the decision whether to serve foreign markets in the first place and the choice whether to invest at home or abroad. In all these contributions, monetary policy is exogenous and firms set prices at the beginning of each period. As a consequence, the only way monetary policy can affect price and entry dynamics is through the exchange rate. The model in this paper removes both those limiting assumptions.


\textsuperscript{2}In many models with endogenous varieties, producer inflation acts as a tax on firms’ profits.
This paper introduces endogenous producer entry into the model of Benigno and Benigno (2008) so as to provide a first step towards the joint determination of exchange rates and product varieties under interest rate rules. The monetary regime is represented in the standard form of a feedback rule as in the Neo-Wicksellian framework (Woodford (2003)). The global nature of the monetary regime is captured by the interaction of interest rate rules followed by the monetary authorities in both countries. I consider two floating regimes, one with symmetric Taylor rules and the other with interest smoothing, and a regime where the exchange rate is fixed at all dates and compare the macroeconomic dynamics that would prevail with flexible prices (i.e., with the Wicksellian policy) with the one that arises under the alternative monetary rules. Nominal rigidity is incorporated through Calvo pricing.

The introduction of endogenous product variety has a number of interesting consequences for the international monetary transmission. First, it affects the dynamics of the exchange rate. In the Wicksellian scenario, the nominal exchange rate follows the path of the terms of trade and the domestic currency appreciates whenever home productivity rises or the number of home varieties declines. Since the source of perturbation in the exchange rate is given by real shocks, either productivity or entry shocks, drawn from stationary distributions, the Wicksellian exchange rate is stationary. Under floating regimes with nominal rigidity, on the contrary the exchange rate is in general non-stationary. Only monetary rules that target the exchange rate manage to stabilize its long-run value. Moreover, the nominal exchange rate also reacts to nominal shocks whenever price adjustment is incomplete. Numerical examples show that even a purely transitory drop in the interest rate is able to generate a persistent depreciation of the domestic currency.

Second, endogenous entry allows to contemplate a novel channel of monetary transmission which is absent in fixed-variety models. As already stressed in closed-economy frameworks, monetary policy can influence entry behavior by changing the perception on the part of potential investors of either the real cost of entry or the expected revenue from their investments (in equilibrium, a free-entry condition must be satisfied requiring that entry costs are equalized to the value of the firm). Bilbiie, Ghironi and Mélitz (2007) stress that monetary authorities, by setting interest rates, can affect the real return on financial assets and the price of equity. In their model, a drop in the nominal interest rate hikes the real cost of equity and therefore deters start-up investments. Others argue that a monetary easing will create a more favorable environment for potential investors by increasing the present value of expected profits, as in Bergin and Corsetti (2005) and Lewis (2006), or by directly reducing the entry fees, as in Elkouri and Mancini Griffoli (2006). In my open-economy setup, an expansion in the global monetary stance has the potential to attract or deter new investments depending on which one between two opposing effects prevails. On the one side,
the drop in the world interest rate makes it more convenient consuming today, thereby reducing
the resources currently available for investments (there is a trade-off between consumption and
investment). On the other side, however, the favorable demand conditions in global markets tend to
attract potential investors. Numerical examples show that the first effect will prevail whenever the
monetary expansion originates at home. The result is due to the incentive on the part of domestic
producers to raise the relative price of their own variety when the domestic currency depreciates.
The opposite might occur with an appreciation of the domestic currency as in the aftermath of a
monetary expansion abroad.

Finally, I stress the implications of endogenous variety for the Phillips curve. In my model,
producer inflation is linked to its expected value and to an index of current marginal costs as
in a typical new-Keynesian Phillips curve. The number of producers featured in current market
conditions is a state variable which contributes to generate endogenous inflation persistence as in
the data. A further difference with the traditional fixed-variety Phillips curve regards the inflationary
consequences of an increase in world demand. As long as the demand boost can be accommodated
only along the intensive margin, i.e. raising output per firm, marginal costs and therefore inflation
will inevitably rise. This is no longer the case when investments can grow along the extensive margin
as well. I show that a rise in the number of producers leads to a fall in inflation as a consequence
of tighter competition in goods markets. These observations have implications for empirical studies
of the Phillips curve. Estimating the curve without accounting for variable varieties along the cycle
raises a potential problem of identification. The observation of a fall in inflation for a given level of
output would in fact be identified as a (positive) cost push shock while it is virtually indistinguishable
from an endogenous adjustment in variety.

Numerical simulations show that the responses to aggregate productivity and entry shocks are
very similar to those in the flexible price benchmark for plausible specifications of interest rate
setting. I stress that Taylor rules with only a limited degree of aggressiveness towards producer
inflation manage to induce a pro-cyclical entry behavior consistently with stylized facts. The cyclical
properties of endogenous variables with sticky prices are in general close to their flexible price
counterpart and close to those in the standard real business cycle model. Major differences concern
the responses of markups and the magnitude of the entry effect. In order to see the point, consider
productivity shocks as the main source of fluctuations. The responses of potential entrants with
sticky prices are subdued relative to those under flexible prices (between two and three times smaller
depending on the monetary rule in place). This is a consequence of nominal rigidity translating into
persistent real rigidity in my setup with endogenous entry. Incomplete price adjustment, in fact,
implies a real price distortion in favour of those firms that can actually change the price of their
product. The possibility of incurring in price distortions, in turn, deters investments and leads to a gradual fall (or to a subdued increase) in the number of producers over time. I stress that the introduction of nominal rigidity acts as sand in the wheels of the entry mechanism and helps alleviate the problem of excess volatility in investments typical of real business cycle models without costs of capital adjustment. Moreover, the real price distortion translates into variable margin of profits over the cycle. Remarkably, the responses of markups are counter-cyclical as in the data.

The paper is organized as follows. Section 2 presents the model and the equilibrium conditions. Section 3 provides the log-linear model and section 4 presents numerical examples. Section 5 concludes.

2 The model

The world economy comprises two countries labelled Home, H, and Foreign, F, each specialized in the production of one type of good as in Corsetti and Pesenti (2002). Each country is populated by a continuum of agents of unit mass. A typical agent in the economy is both a consumer and a worker: he supplies labor services in a competitive labor market and consumes all the goods produced in the world economy. In the Home country, there is a continuum of monopolistically competitive firms, each producing a different variety of the Home good $h \in (0, N^H)$, where $N^H$ is the number of Home firms. Similarly, in the foreign country there is a continuum of firms $f \in (0, N^F)$. The stock of producers in the Home and Foreign countries is determined endogenously in the model.

2.1 Consumers’ preferences

A typical agent $i$ in country $J = H, F$ derives utility from consuming a composite index of all the goods produced in the world economy, $C$, and from holding real money balances, $\frac{M}{P}$, while derives disutility from labour effort, $L$. Agents maximize the expected discounted value of flow utility $U$ over their life horizon. Flow utility is assumed additive-separable:

$$U^J_i(t) = \left(\frac{C^J_i(t)}{1 - \rho}\right)^{1 - \rho} + \left(\frac{M^J_i(t)}{P^J_i}\right)^{1 - \nu} - \frac{\varphi \chi}{1 + \varphi} \left(\frac{L^J_i(t)}{1 + \varphi}\right)^{1 + \varphi}$$

where $\rho > 0$ is the inter-temporal elasticity, $\nu > 0$ the elasticity of real money balances and $\varphi > 0$ is the Frisch elasticity of labour supply.

The consumption basket $C$ comprises home, $C_H$, and foreign goods, $C_F$. 

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3Unless otherwise stated, nominal variables are expressed in domestic currency. So $M^F$, for instance, denotes the amount of money supply in foreign currency.
\[ C^J = \frac{(C^J_H)^\gamma (C^J_F)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}} \]  \hspace{1cm} (2)

where \( C_H \) and \( C_F \) are given by:

\[ C^J_H = \left[ \int_0^{N_H} C^J(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{(\theta-1)}} \]

\[ C^J_F = \left[ \int_0^{N_F} C^J(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{(\theta-1)}} \]  \hspace{1cm} (3)

The consumer price index is defined as:

\[ P^J = (P^J_H)^\gamma (P^J_F)^{1-\gamma} \]  \hspace{1cm} (4)

where

\[ P^J_H = \left[ \int_0^{N_H} P^J(h)^{1-\theta} dh \right]^{\frac{1}{(1-\theta)}} \]  \hspace{1cm} (5)

\[ P^J_F = \left[ \int_0^{N_F} P^J(f)^{1-\theta} df \right]^{\frac{1}{(1-\theta)}} \]

I assume that prices are set in the producers’ currency and that the law of one price holds \( P^H(h) = \varepsilon P^F(h) \) and \( P^H(f) = \varepsilon P^F(f) \), where \( \varepsilon \) is the nominal exchange rate defined as the price of currency F in terms of currency H. Notice that overlooking segmentation in international goods markets does not imply a loss of generality in my setup where all goods are traded. The currency of denomination of international trade might play a role in the decision whether to serve foreign customers in the first place and eventually whether to serve them with exports or by engaging in investments overseas.

The analysis of endogenous changes in trade openness or in the mode of accessing foreign markets is beyond the scope of the present paper.  

Finally, I define the terms of trade of country F, \( T \), as the price of the bundle of goods produced in country F relative to the price of the bundle of goods produced in country H. Therefore, \( T = P^F_H / P^H_F = P^F_H / P^H_H \).

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\(^4\)Cavallari (2009) provides a model with endogenous trade openness and endogenous entry of export and multinational firms. The currency of denomination of international trade is found to affect both dimensions of the decision to serve foreign markets.
2.2 Firms

Producers in the world economy face an identical linear technology where labor is the sole factor. Output supplied by a firm \( h \) in country \( H \) and a firm \( f \) in country \( F \) is given by, respectively:

\[
y_t^H(h) = Z_t^H L_t^H(h) \quad y_t^F(f) = Z_t^F L_t^F(f) \tag{6}
\]

where \( Z^J \) is aggregate labor productivity in country \( J = H, F \). Productivity is exogenous and follows an AR(1) process in percent deviations from the steady state.

Prior to entry, firms face a sunk entry cost \( f^{EJ} \) defined in units of consumption: entry requires purchasing a basket of materials of amount \( f^{EJ} \) which has the same composition as the consumption basket, \( C \). Others, as Bilbiie, Ghironi and Méitz (2007), Bergin and Corsetti (2005) and Cavallari (2007) among others, define entry costs in effective labor units. Entry in this case requires hiring a certain amount of workers. Notice that the sunk nature of the cost implies that prior to entry firms need to devolve a certain amount of resources to start-up activities independently on whether these resources are expressed in consumption or in labor units.\(^5\) The normalization clearly has implications for the definition of aggregate variables. With entry costs in units of consumption, for instance, output of the consumption sector coincides with GDP while this is no longer the case when entry costs are in labor units.

The entry cost is exogenous and follows an AR(1) process in percent deviations from the steady state. All firms entered in a given period are able to produce in all subsequent periods until they are hit by a death shock, which occurs with a constant probability \( \delta \in (0, 1) \). Therefore, a firm entered in period \( t \) will only start producing at time \( t + 1 \), introducing a one-period time-to-build lag into the model.

In each period, there is a finite mass of potential entrants, \( N^{EJ} \). Entrants are forward looking and decide to start a new production line whenever the real value of the new firm, \( \nu \), covers entry costs. Therefore, the free entry condition is given by:

\[
\nu_t^J = E_t \left[ \sum_{s=t+1}^{\infty} \beta (1 - \delta) \left( \frac{C_t^{J+1}}{C_t^{J}} \right)^{-\rho} \right] = f_t^{EJ} \tag{7}
\]

where the expression in square brackets is the present discounted value of future real profits, \( d \).

Finally, the timing of entry implies the following law of motion for the number of producing firms:

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\(^5\)One can think of sunk entry costs as administrative fees, licences or any other bureaucratic accomplishment which is required before entering the market or as the cost for advertising the launch of new products.
\[ N_t^J = (1 - \delta) \left( N_{t-1}^J + N_{t-1}^{E_J} \right) \]

### 2.3 Consumers’ choices

I assume that financial markets are complete within and across countries by allowing agents to trade in a set of nominal state-contingent bonds denominated in the currency of country \( J = H, F, B^J \), that span all the states of nature.\(^6\) In addition to state-contingent bonds, a typical agent holds domestic currency and a share \( s \) of a well-diversified portfolio of domestic firms. His budget constraint is given by:

\[ \sum_{\Omega} q^J(\Omega_{t+1}) \frac{B^J_{it}}{P_t^J} + \frac{M^J_{it}}{P_t^J} + s^J_t (N_t^J + N_{t-1}^{E_J}) v^J_t \leq \frac{B^J_{it-1}}{P_t^J} + \frac{M^J_{it-1}}{P_t^J} + s^J_{t-1} (v_t^J + d_t^J) \]

\[ + \frac{W^J_t L^J_{it}}{P_t^J} - C^J_{it} + \frac{TR^J_{it}}{P_t^J} \]

where \( W \) is the nominal wage and \( TR \) is a nominal transfer from the government.

Agents choose consumption, labor effort, money, share and bond holdings in period \( t \) so as to maximize utility (1) over their whole life horizon subject to the budget constraint (8). Consumers’ optimization requires the following first order conditions:

\[ \frac{q^J_t (s_{t+1})}{P_t^J} C_t^{-\rho} = \beta E_t \left( \frac{C_{t+1}^J}{P_{t+1}^J} \right)^{-\rho} \]

(9)

\[ (C_t^J)^{-\rho} = \beta (1 - \delta) E_t \left[ \frac{d^J_{t+1} + v^J_{t+1} C_{t+1}^{-\rho}}{v_t^J} \right] \]

(10)

\[ (C_t^J)^{-\rho} = \lambda_t P_t^J \]

(11)

\[ C_t^J(h) = \left( \frac{P_t^J(h)}{P_{ht}^J} \right)^{-\theta} C_{ht}^J \]

(12)

\[ C_t^J(f) = \left( \frac{P_t^J(f)}{P_{ft}^J} \right)^{-\theta} C_{ft}^J \]

(13)

\[ \left( \frac{M^J_{it}}{P_t^J} \right)^{\nu} = \frac{(C_t^J)^{\rho}}{1 - (1 + i_t^J)^{-1}} \]

(14)

\(^6\)There exists a complete set of bonds which pay one unit of the domestic currency if state \( \Omega_{t+1} \) occurs at time \( t+1 \). The price of such a bond at date \( t \) is \( q^J(\Omega_{t+1}) \).
\[ \frac{W_t^J}{P_t^J} = (L_t^J)^{1/\rho} (C_t^J)^\rho \] (15)

Notice that the Euler equation for bonds (9) plus the no arbitrage condition \( q_t^H (s_{t+1}) = \varepsilon_t q_t^F (s_{t+1}) \), yields the risk-adjusted uncovered interest rate parity:

\[ E_t \left( \frac{P_t^H C_t^{-\rho}}{P_{t+1}^H C_{t+1}^{-\rho}} \right) = E_t \left( \frac{P_t^F C_t^{-\rho}}{P_{t+1}^F C_{t+1}^{-\rho}} \right) \left( 1 + \frac{i_{t+1}^F}{1 + i_{t+1}^H} \right) \] (16)

Finally, I stress that the assumption of complete markets together with the law of one price and the fact that consumption bundles are identical across countries imply that consumption risks are fully insured worldwide, i.e. \( C_t^H \equiv \int_0^1 C_i^H \, di = C_t^F \equiv \int_0^1 C_i^F \, di \).

### 2.4 Pricing

Goods markets are monopolistically competitive. Each producer sets the price for its own variety facing market demand:

\[ y_t(h) = \left( \frac{P_t^J(h)}{P_{t+1}^H} \right)^{1/\gamma} T_t \]  

\[ y_t(f) = \left( \frac{P_t^J(f)}{P_{t+1}^F} \right)^{1/\gamma} T_t \] (17)

I introduce nominal rigidities through a Calvo-type contract. In each period a firm can set a new price with a fixed probability \( 1 - \alpha \) which is the same for all firms and is independent on the time elapsed since the last price change. In every period there will therefore be a share \( \alpha \) of firms whose prices are pre-determined.

A firm sets the price for its own variety so as to maximize the expected discounted value of profits, taking into account market demand and the probability that she might not be able to change the price in the future, yielding:

\[ P_t^H(h) = \frac{\theta}{\theta - 1} \left( 1 - \alpha \beta (1 - \delta) \right) \frac{W_t^H}{Z_t^H} + \alpha \beta (1 - \delta) E_t P_{t+1}^H(h) \]  

\[ P_t^F(f) = \frac{\theta}{\theta - 1} \left( 1 - \alpha \beta (1 - \delta) \right) \frac{W_t^F}{Z_t^F} + \alpha \beta (1 - \delta) E_t P_{t+1}^F(f) \] (18)

The optimal price depends on current nominal marginal costs as well as on the expectation of future price changes. Clearly, when \( \alpha = 0 \) the optimal price implies a constant markup \( \frac{\theta}{\theta - 1} \) on nominal marginal costs.

Calvo price-setting implies the following state equations for \( P_t^H \) and \( P_t^F \):

\[ \text{...} \]
\[(P^H_{ht})^{1-\theta} = \frac{N^H_t}{N^H_{t-1}} (P^H_{ht-1})^{1-\theta} + \alpha N^H_t (P^H_t(h))^{1-\theta}\]
\[(P^F_{ft})^{1-\theta} = \frac{N^F_t}{N^F_{t-1}} (P^F_{ft-1})^{1-\theta} + \alpha N^F_t (P^F_t(f))^{1-\theta}\]

Observe that producer prices depend on the stock of current and past producers in this setup with endogenous entry.

### 2.5 Aggregate accounting

Governments in the two countries simply rebate all seigniorage revenue in lump-sum transfers to households:

\[
\int_0^1 M^H_t - M^H_{t-1} di + \int_0^1 TR^H_t di = 0 \quad \int_1^2 M^F_t - M^F_{t-1} di + \int_1^2 TR^F_t di = 0
\]

Labor market clearing implies:

\[
L^H_t = \int_0^1 L^H_t di \geq \int_0^{N^H_t} \frac{y_t(h)}{Z^H_t} dh \quad L^F_t = \int_1^2 L^F_t di \geq \int_0^{N^F_t} \frac{y_t(f)}{Z^F_t} df
\]

Aggregating the budget constraint (8) across agents, using the government (20) and resource constraint (21) and imposing the equilibrium conditions \(s^J_{t+1} = s^J_t = 1\) and \(B^J_{t+1} = B^J_t = 0\) for any \(t\) yields the aggregate accounting equation:

\[
Y^J_t \equiv C^J_t + N^E^J v^J_t = \frac{W^J_t L^J_t}{P^J_t} + d^J_t N^J_t
\]

where \(Y^J\) is GDP in country \(J = H, F\): consumption plus investments in new firms must be equal to income (wages plus profits).

The equilibrium conditions of the model are summarized in Table 1.
Table 1: model summary

| Free entry | \( \nu_t = f_t^{EJ} \) |
| Number of firms | \( N_t^J = (1 - \delta) (N_{t-1} + N_t^{EJ}) \) |
| Intra-temporal optimality | \( C^J(h) = \left( \frac{P^J(h)}{P^H(h)} \right)^{-\sigma} C^J_H \) |
| Euler equation (shares) | \( C_t^{-\rho} = \beta (1 - \delta) E_t \left[ \frac{d_{t+1} + v_{t+1}}{v_t} C_{t+1}^{-\rho} \right] \) |
| Euler equation (bonds) | \( g_t^{(s_{t+1})} C_t^{-\rho} = \beta E_t \left( \frac{C_{t+1}}{P_{t+1}} \right)^{-\rho} \) |
| Labor supply | \( W_t = \frac{1}{\beta} C_t^\rho \) |
| Output per firm | \( y(h) = \left( \frac{P^H(h)}{P^H_*} \right)^{-\sigma} T^{1-\gamma} C \) |
| Aggregate output | \( Y^H \equiv \int_h^{N_t^J} \left( \frac{P^H(h)}{P^H_*} \right) y(h) dh \) |
| Aggregate accounting | \( Y^F \equiv \int_h^{N_t^{EF}} \left( \frac{P^F(h)}{P^F_*} \right) y(f) df \) |
| Aggregate output | \( Y^J = C_t^J + N_t^{EJ} v_t^J = \frac{W_t^J I_t^J}{P_t^J} + d_t^J N_t^J \) |

2.6 Interest rate rules

The model is closed by specifying the monetary policy rules in place in the world economy. I assume that the monetary instrument is the one-period risk-free nominal interest rate, \( i_t^J \), and consider the class of feedback rules:

\[
1 + i_t^J = f_t^J(\Theta_t) \tag{23}
\]

where \( f \) is a generic function and \( \Theta \) is the information set at time \( t \).

2.7 Equilibrium

The model is not solvable in closed form. In the remaining I will focus on equilibria where the state variables follow paths that are close to a symmetric deterministic stationary equilibrium in which producer price inflation and exchange rate depreciation are zero. In this steady state the stochastic shocks are muted at all dates, \( Z^J = f^E = 1 \). The steady state of the model is summarized in Table 2.

11
Table 2: symmetric steady state

| Interest rate | \( i = \frac{1-\beta}{\beta} \) |
| Terms of trade | \( T = 1 \) |
| Nominal exchange rate | \( \varepsilon = 1 \) |
| Consumption | \( C = \theta N \left[ \frac{1-\beta(1-\delta)}{\beta(1-\delta)} - \frac{\delta}{\theta(1-\delta)} \right] \) |
| Number of firms | \( N = \frac{1-\delta}{\delta} N^E \) |
| Number of entrants | \( N^E = \left( \frac{\theta}{\theta - 1} \right)^{\varphi} N^{1+\varphi} C^{-\varphi \rho} - C \) |
| Employment | \( L = \left( \frac{\theta}{\theta - 1} \right)^{-\varphi} N^{\frac{\varphi}{1+\varphi}} Z^{\varphi} C^{-\varphi \rho} \) |
| GDP | \( Y = \left( \frac{\theta}{\theta - 1} \right)^{-\varphi} N^{\frac{\varphi}{1+\varphi}} Z^{1+\varphi} C^{-\varphi \rho} \) |
| Profits | \( d = \frac{1-\beta(1-\delta)}{\beta(1-\delta)} \) |

Notice that an increase in the stock of production lines, \( N \), leads to a more than proportional increase in consumption. This is a consequence of love for variety, namely the fact that agents derive more utility from spanning a given change in consumption over a larger bundle of goods than consuming a larger amount of each variety. I further stress the existence of a trade-off between investments and consumption. As steady state consumption rises, in fact, aggregate accounting implies that fewer resources will be available for investments. Consequently, \( N^E \) must fall.

3 The log-linear model

3.1 Demand block

The aggregate demand block is derived from the log-linear approximation to the first order conditions of the consumers in the Home and Foreign countries. Consumers allocate their wealth among consumption, nominal risk-free securities and shares. The first-order conditions of inter-temporal optimization require that the marginal rate of substitution between current and one-period ahead consumption must equalize the real return on nominal assets, both the risk-free bonds and shares. A first set of Euler equations, one for each country, will therefore describe the dynamic link between current and expected one-period ahead consumption and relate it to the risk-free return in units of consumption. A second set of Euler equations, again one for each country, will instead relate the inter-temporal profile of consumption to the real return on shares. The real value of the firm, which coincides with the exogenous entry shock in this model, is the forward solution of the Euler equations for shares.

The assumption of complete asset markets, together with the law of one price and the fact that
the consumption index is identical across countries, implies that consumption risks are perfectly insured in the world economy. I can therefore combine the bond Euler equations in the Home and Foreign countries, yielding:

\[ E_t \hat{C}_{t+1} = \hat{C}_t + \frac{\gamma}{\rho} \left( \hat{i}_t^H - E_t \pi_{t+1}^H \right) + \frac{1 - \gamma}{\rho} \left( \hat{i}_t^F - E_t \pi_{t+1}^F \right) \]  

(24)

where a hat over a variable denotes the log-deviation from the steady state and \( E_t \pi_{t+1}^J \) is the expected one-period ahead producer inflation in country \( J = H, F \). An increase in the world real interest rate, wherever it is originated, raises the return on bonds, therefore making it more attractive to postpone consumption in the future.

By taking the difference between the bond Euler equations across countries, I get a log-linear approximation to the uncovered interest parity, which links the expected depreciation rate of the domestic currency to the interest rate differential:

\[ E_t \Delta \hat{\varepsilon}_{t+1} = \hat{\gamma}_t^H - \hat{\gamma}_t^F \]  

(25)

Finally, I take a log-linear approximation to the definition of the terms of trade, obtaining:

\[ \hat{T}_t = \hat{T}_{t-1} + \Delta \hat{\varepsilon}_t + \pi_t^F - \pi_t^H \]  

(26)

Notice that movements in the terms of trade around the steady state depend on exchange rate fluctuations as well as on producer inflation in the two countries.

### 3.2 Supply block

The aggregate supply block is derived from the log-linear approximation to the optimal pricing and entry decisions of the firms in the Home and Foreign countries. First, consider the Phillips curves:

\[ \pi_t^H = \frac{(1 - \alpha \beta (1 - \delta)) (1 - \alpha) \varphi \hat{m}_t^H + \beta (1 - \delta) E_t \pi_{t+1}^H}{\alpha (\varphi + \theta)} \]  

(27)

\[ \pi_t^F = \frac{(1 - \alpha \beta (1 - \delta)) (1 - \alpha) \varphi \hat{m}_t^F + \beta (1 - \delta) E_t \pi_{t+1}^F}{\alpha (\varphi + \theta)} \]

where \( \hat{m}_t^J \) is an index of real marginal costs in country \( J = H, F \) at time \( t \). Marginal costs are positively correlated with a rise in world consumption and a fall in the relative price of domestic products, while they are negatively associated with productivity shocks and the growth in the number
of producers. The former two variables capture the inflationary effect of an increase in aggregate demand accommodated along the intensive margin, i.e. with higher output per firm. When output increases along the extensive margin, on the contrary, inflationary pressure reduces as a consequence of tighter competition among producers. I stress that in my setup with endogenous entry, the new-Keynesian Phillips curves (27) feature a state variable, the number of producers, which introduces an endogenous persistence into the pattern of inflation as in the data. Furthermore, the presence of endogenous variety has relevant implications for empirical studies of the Phillips curve. Estimating the curve without accounting for variable varieties along the cycle raises a potential problem of identification. The observation of a fall in inflation for a given level of output would in fact be identified as a (positive) cost push shock while it is virtually indistinguishable from an endogenous adjustment in variety.

Second, I specify an aggregate production function in the Home and Foreign countries

\[ Y_t^H = \int h^N_H \frac{P^H(h)}{P^H} y(h) dh \quad \text{and} \quad Y_t^F = \int h^N_F \frac{P^F(f)}{P^F} y(f) df, \]

where the relative price \( P^H(h) \) is included so as to convert output in terms of the consumption good. Note that with Calvo pricing, firms will produce different amounts of output at any given time depending on whether they will be able to change the price of their own products. A log-linear approximation to these definitions, after some manipulations, yields:

\[
\hat{Y}_t^H = (1 + \varphi)(\gamma - 1) \hat{T}_t + (1 + \varphi) Z_t^H - \varphi \rho \hat{C}_t + \left( \frac{1 + \varphi}{(\theta - 1)(1 - \alpha)} \right) \hat{N}_t^H + \\
\frac{\alpha (1 + \varphi)}{1 - \alpha} \hat{N}_{t+1}^H - \frac{\alpha (1 + \varphi)}{(\theta - 1)(1 - \alpha)} \hat{N}_{t-1}^H
\]

\[
\hat{Y}_t^F = -\gamma (1 + \varphi) \hat{T}_t + (1 + \varphi) Z_t^F - \varphi \rho \hat{C}_t + \left( \frac{1 + \varphi}{(\theta - 1)(1 - \alpha)} \right) \hat{N}_t^F + \\
\frac{\alpha (1 + \varphi)}{1 - \alpha} \hat{N}_{t+1}^F - \frac{\alpha (1 + \varphi)}{(\theta - 1)(1 - \alpha)} \hat{N}_{t-1}^F
\]

The first three terms in the expressions above capture the internal margin of output fluctuations. Focusing on Home output, both an improvement in the home terms of trade, an increase in home productivity and a fall in consumption lead to a higher real aggregate wage in equilibrium and therefore to higher employment (recall labor supply (15)). For a given number of producers, this

\[ \tilde{m}_c^H = \frac{(1 - \gamma)(1 + \varphi)}{\varphi} \hat{T}_t + \left( \rho + \frac{1}{\varphi} \right) \hat{C}_t - \frac{1}{(1 - \alpha)(\theta - 1)} \hat{N}_t^H - \frac{(1 + \varphi)}{\varphi} Z_t^H + \frac{\alpha}{(1 - \alpha)(\theta - 1)} \hat{N}_{t-1}^H \]

\[ \tilde{m}_c^F = -\frac{\gamma (1 + \varphi)}{\varphi} \hat{T}_t + \left( \rho + \frac{1}{\varphi} \right) \hat{C}_t - \frac{1}{(1 - \alpha)(\theta - 1)} \hat{N}_t^F - \frac{(1 + \varphi)}{\varphi} Z_t^F + \frac{\alpha}{(1 - \alpha)(\theta - 1)} \hat{N}_{t-1}^F \]
in turn implies that firms’ size will be larger. The last three terms capture the effects of price distortions due to Calvo pricing. The fact that the relative price $P^H(h)/P^H$ will vary across firms has two implications for output. First, it affects the value of output in terms of consumption. In an inflationary environment it is easier to raise the relative price of a single variety (for those firms that can actually do it), thereby increasing the value of output. This explains the positive relation between changes in output and inflation. Second, relative price distortions affect the external margin of output fluctuations in a way that will be evident soon. At this stage, I only stress that accounting for endogenous movements in the stock of producers introduces persistence into the dynamics of output.

Third, I derive a log-linear approximation to the number of entrants from the aggregate accounting relation, obtaining:

$$\hat{N}^E_J = -J^E_J + \frac{\theta (1 - \beta (1 - \delta))}{\beta \delta} \tilde{y}^J_t + \left(1 - \frac{\theta (1 - \beta (1 - \delta))}{\beta \delta}\right) \tilde{c}_t$$

(29)

Finally, the law of motion of firms is:

$$\hat{N}^J_t = (1 - \delta) \hat{N}^J_{t-1} + \delta \hat{N}^E_J$$

(30)

### 3.3 Interest rate rules

I consider one regime with fixed exchange rates and two floating regimes.

The first regime is a unilateral peg to the Foreign currency and features a fixed exchange rate at all dates. There is a potential problem of exchange rate indeterminacy. In order to see why, consider that exchange rate expectations are zero whenever the Home and Foreign nominal interest rates are equalized, as it appears from uncovered interest parity (25). The rule $\tilde{r}^H_t = \tilde{r}^F_t$, however, leaves the level of the nominal exchange rate indetermined. As outlined in Benigno and Benigno (2008), there is a simple way of overcoming the problem: the Home nominal interest rate is set so as to follow the Foreign interest rate and react to the deviation of the exchange rate from a desired target (here, normalized to zero), i.e. $\tilde{r}^H_t = \tilde{r}^F_t - \zeta \tilde{e}_t$ for any $\zeta > 0$.

The second regime features a symmetric Taylor rule where the nominal interest rate in each country reacts to producer inflation, $\tilde{r}^I_t = \phi_t^I \pi_t^I$. Taylor rules have been extensively analyzed since the seminal paper by Taylor (1993). They do a surprisingly good job of capturing the policy of a large number of central banks especially in the last few decades when the objective of price stability has gained a major role in monetary policy-making. As shown in Benigno and Benigno (2008), the Taylor principle, requiring that policy-makers react more than proportionally to inflation, i.e.
Finally, I consider a smoothing regime where nominal interest rates depend on their past values. In particular, I focus on smoothing Taylor rules as \( \hat{i}_t^j = \phi_h \hat{i}_{t-1}^j + \phi_p \pi_t^j \). The rationale for smoothing rules draws on the desire to reduce volatility in nominal interest rates which might translate into real volatility given the long and variable lags in monetary transmission.

3.4 Equilibrium with flexible prices

In any equilibrium with flexible prices, only real shocks affect real variables. The patterns of real variables are given in Table 3:

<table>
<thead>
<tr>
<th>Table 3: flexible price equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption</strong></td>
</tr>
<tr>
<td>( \tilde{C}<em>t = \frac{1}{(1 + \varphi)} \left[ \tilde{N}</em>{t}^{W} + (1 + \varphi) Z_{t}^{W} \right] )</td>
</tr>
<tr>
<td><strong>Terms of trade</strong></td>
</tr>
<tr>
<td>( \tilde{T}<em>t = \frac{1}{(1 + \varphi)} \left[ \tilde{N}</em>{t}^{R} - Z_{t}^{R} \right] )</td>
</tr>
<tr>
<td><strong>Home GDP</strong></td>
</tr>
<tr>
<td>( \tilde{Y}<em>t^H = (1 + \varphi)(\gamma - 1) \tilde{T}<em>t + (1 + \varphi) Z</em>{t}^{H} + \frac{(1 + \varphi)}{\varphi - 1} \tilde{N}</em>{t}^{H} - \rho \varphi \tilde{C}_t )</td>
</tr>
<tr>
<td><strong>Foreign GDP</strong></td>
</tr>
<tr>
<td>( \tilde{Y}<em>t^F = \gamma (1 + \varphi) \tilde{T}<em>t + (1 + \varphi) Z</em>{t}^{F} + \frac{(1 + \varphi)}{\varphi - 1} \tilde{N}</em>{t}^{F} - \rho \varphi \tilde{C}_t )</td>
</tr>
<tr>
<td><strong>Number of firms</strong></td>
</tr>
<tr>
<td>( \tilde{N}<em>{t+1}^J = (1 - \delta) \left[ \tilde{N}</em>{t+1}^J + \delta \tilde{N}_{t}^{JE} \right] )</td>
</tr>
<tr>
<td><strong>Number of entrants</strong></td>
</tr>
<tr>
<td>( \tilde{N}<em>{t+1}^{JE} = -f</em>{t}^{JE} + \frac{\theta(1-\beta)(1-\delta)}{\beta \delta} \tilde{Y}_{t}^J - \frac{\theta(1-\beta)(1-\delta) - \beta \delta}{\beta \delta} \tilde{C}_t )</td>
</tr>
<tr>
<td><strong>Variety effect</strong></td>
</tr>
<tr>
<td>( \frac{\tilde{P}<em>{t}^{(h)}}{\tilde{P}</em>{t}^{H}} = \frac{1}{(\varphi - 1)} \tilde{N}<em>{t}^{H} ) ( \frac{\tilde{P}</em>{t}^{(f)}}{\tilde{P}<em>{t}^{F}} = \frac{1}{(\varphi - 1)} \tilde{N}</em>{t}^{F} )</td>
</tr>
<tr>
<td><strong>Relative price</strong></td>
</tr>
<tr>
<td>( \frac{\tilde{P}<em>{t}^{(h)}}{\tilde{P}</em>{t}^{H}} = \frac{1}{(\varphi - 1)} \tilde{N}<em>{t}^{H} + (\gamma - 1) \tilde{T}<em>t ) ( \frac{\tilde{P}</em>{t}^{(f)}}{\tilde{P}</em>{t}^{F}} = \frac{1}{(\varphi - 1)} \tilde{N}_{t}^{F} + \gamma \tilde{T}_t )</td>
</tr>
<tr>
<td><strong>Real equity price</strong></td>
</tr>
<tr>
<td>( \tilde{v}<em>{t}^{J} = f</em>{t}^{JE} - \frac{\tilde{P}<em>{t}^{(h)}}{\tilde{P}</em>{t}^{H}} )</td>
</tr>
</tbody>
</table>

where a tilde over a variable denotes the flexible price value (in log deviation) and for any variable \( X, X_t^R \equiv X_t^H - X_t^F \) while \( X_t^W \equiv (X_t^H)_{\gamma}(X_t^F)^{1-\gamma} \). Only world productivity shocks and changes in the world stock of producers affect consumption, while relative shocks and changes in the relative number of producers affect the terms of trade.

Monetary policy shocks affect nominal variables. A given change in world nominal interest rates, however, might be associated with multiple paths for the nominal exchange rate and producer price inflation (recall the terms of trade equation (26) showing that any given change in \( \tilde{T} \) is consistent with multiple combinations of \( \Delta \varepsilon \) and \( \pi^J \)). In what follows, I will draw the attention on equilibria where producer price inflation is zero. In a setup with fixed entry, the flexible price equilibrium with zero inflation corresponds to the policy that would be chosen by a central planner interested in maximizing the utility of all agents in the world economy. \(^8\) As already outlined by Bilbiie, Ghironi

\(^8\)See, for instance, Benigno and Benigno (2008). In open economies, a terms of trade externality might exist, implying that an improvement in the terms of trade of, say, H consumers might increase their level of consumption for
and Mélitz (2007), the same holds in a setup with endogenous entry where producer price inflation distorts firm entry decisions and therefore the allocation of resources between the creation of new varieties and the production of existing goods.

Under the assumption of zero producer price inflation, the nominal exchange rate directly follows the paths of the terms of trade. Given the initial conditions \( \tilde{T}_{-1} = \tilde{e}_{-1} = 0 \), equation (26), in fact, implies that \( \tilde{T}_t = \tilde{e}_t \) at all dates. Combining uncovered interest parity (25) and the Euler equations in the two countries yields the optimal or Wicksellian interest rates:

\[
\tilde{i}_{t}^{H} = \rho \left( E_{t} \bar{C}_{t+1} - \bar{C}_{t} \right) + (1 - \gamma) \left( E_{t} \tilde{T}_{t+1} - \tilde{T}_{t} \right) \\
\tilde{i}_{t}^{F} = \rho \left( E_{t} \bar{C}_{t+1} - \bar{C}_{t} \right) - \gamma \left( E_{t} \tilde{T}_{t+1} - \tilde{T}_{t} \right)
\]

As well-known, the optimal policy can be easily implemented recurring to a credible threat to deviate from the zero inflation target, i.e. policy-makers follow the rule \( i_{t}^{d} = \tilde{i}_{t}^{d} + \vartheta \pi_{t}^{d} \) for any \( \vartheta > 0 \).

4 Results

In this section I explore the properties of the model presented above by means of numerical examples. I compute impulse responses to entry, productivity and monetary policy shocks that originate in the Home and Foreign countries.

4.1 Calibration

I calibrate a US-EMU model in which country H is the US and country F the EMU. As both countries contribute to world production in approximately the same way, I consider their size to be equal and set \( \gamma = 0.5 \). In the baseline calibration, periods are interpreted as quarters and \( \beta = 0.99 \) as usual in quarterly models of the business cycle. Following Bilbiie, Ghironi and Mélitz (2007) I set the size of the exogenous exit shock to 0.025. The rate of firm disappearance is consistent with a 10 percent rate of job destruction per year as found in the US. Moreover, such a moderate rate does not overstate the capacity of the model to generate persistence. The elasticity \( \theta \) is set at 7.88 as in Rotemberg and Woodford (1999), resulting in a reasonable average markup of approximately 18 percent. Trade studies based on micro data usually find a much lower \( \theta \). The qualitative features of the impulse responses below do not change if I set \( \theta = 3.88 \).\(^9\) The inter-temporal elasticity \( \rho \)\(^a\) a given level of labor effort. The externality clearly disappears from the point of view of a central planner interested in the utility of all agents in the world economy.

\(^{9}\)Responses with \( \theta = 3.88 \) are available upon request.
and the Frisch elasticity $\varphi$ also follow the parametrization in Rotemberg and Woodford (1999) and are set to, respectively, 0.16 and 2.13. I consider different values for these elasticities and set the weight of labour effort in the utility function so that the steady state level of labour supply is unity irrespective of $\varphi$ (this requires $\chi = 0.9224271$).

The degree of nominal rigidity is taken from Gali, Gertler and Lopez-Salido (2001) who find a value for $\alpha$ in the US between 0.407 and 0.66 while the corresponding value for the EMU is between 0.67 and 0.771. As in Benigno and Benigno (2008), I take the middle points from these intervals and set $\alpha = 0.49$ in the US and $\alpha = 0.72$ in the EMU, implying an average duration of nominal contracts of, respectively 2.3 and 3.65 quarters. I also experiment with a common value for the degree of nominal rigidity in the two countries equal to 0.66 as in Rotemberg and Woodford (1999), obtaining qualitatively identical responses. Initial conditions for productivity and entry shocks as well as for the terms of trade and the nominal exchange rate do not affect the dynamics of the model and can be set at unity without loss of generality.

### 4.2 Results

#### 4.2.1 Entry costs

Figure 1 shows the impulse responses to a 1 percent increase in domestic entry costs. The shock has a persistence of 0.9 as in Elkouri and Mancini-Griñoli (2006). The figure compares the efficient flexible-price equilibrium obtained under the hypothesis that both countries follow a Wicksellian policy (blue line), with three alternative regimes. The first regime is a symmetric Taylor rule where nominal interest rates respond to domestic inflation, $i_H^t = 1.5 \pi_H^t$ in the US and $i_F^t = 1.5 \pi_F^t$ in the EMU (green line); the second is a rule involving interest rate smoothing, which features the same long-run response to inflation, 1.5, as the previous one, $i_H^t = 0.8i_{t-1} + 0.3\pi_H^t$ and $i_F^t = 0.8i_{t-1} + 0.3\pi_F^t$ (black line) and the last is a unilateral peg to the dollar, $i_H^t = 1.5\pi_H^t$ and $i_F^t = i_H^t - 0.2\varepsilon_t$ (red line).

In figure the horizontal axis displays the periods after the shock, while responses denote deviations from steady state (a value of, say, 0.01 indicates a 1 percent deviation).

Consider the responses when the optimal policy is in place. The hike in entry costs, by increasing the real price of investments, temporarily discourages entry, therefore lowering the number of entrants, $N^E$. As typical in RBC models without adjustment costs in capital accumulation, the response of investments concentrates in the earlier periods of the transition, with the number of entrants returning to steady state sooner than other variables, in approximately 20 quarters. Over time, the drop in investments translates into a gradual fall in the number of producers that reaches its peak after 15 periods and then slowly returns to the steady state. It is worth noticing that
on impact adjustment takes place exclusively along the intensive margin, with an increase in output per firm (not shown in Figure 1). As less varieties are produced in the home economy, world consumption reduces. Despite the fall in consumption and the smaller number of entrants, GDP increases on impact and stays above steady state throughout the transition driven by the increase in the real value of investments.\textsuperscript{10} Notably, in my setup with endogenous entry the responses of key macroeconomic aggregates display the typical hump-shape pattern as in the data. Moreover, entry contributes to induce responses that persist well beyond the duration of the shock (around 40 quarters).

Finally, I stress that the Wicksellian interest rate does not respond to a rise in domestic entry costs. This is a consequence of the fact that relative prices in the world economy will automatically and completely adjust through the variety effect: the fall in the number of domestic producers leads to a gradual increase in the relative price of each home variety, thereby improving the home terms of trade and appreciating the domestic currency, $T$ and $\varepsilon$ are below their natural levels throughout the transition. As it will be apparent soon, movements in the terms of trade will transmit the entry shock worldwide.

When comparing responses across policy rules, the results show that the dynamics of macroeconomic aggregates under the three rules considered is strikingly similar to the one in the flexible-price

\textsuperscript{10}With an inter-temporal elasticity equal to 1.16, the fall in aggregate demand prevails on the value effect and GDP falls on impact.
benchmark. The responses of consumption, the terms of trade and the number of entrants are very close indeed. Those of GDP, nominal exchange rates, nominal interest rates, inflation and markups are qualitatively different, although their size is very small. The results are consistent with the findings of Bilbiie, Ghironi and Mélitz (2007) who stress the remarkable similarity of macroeconomic dynamics under optimal and simple monetary rules in a closed-economy setup with endogenous entry.

In order to see why, focus on the path of inflation. In the flexible price scenario, relative prices adjust completely through the variety effect with no need to generate producer inflation. With Calvo pricing, on the contrary, producers anticipate the possibility that they might not be able to change the price of their products in the future and set prices at a premium over marginal costs. The premium (discount) reflects inflationary (deflationary) expectations. I stress that the three monetary rules considered manage to bring inflation expectations quite close to zero (the responses of inflation under the simple rules are below 0.002 per cent). In the early periods after the entry shock, the drop in aggregate demand is associated with decreasing domestic prices, resulting in a very moderate deflation in the home economy. Over time, as demand recovers and producers exit domestic markets, prices gradually begin to grow, drawing inflation above the steady state for a while. The patterns of inflation are very similar across the three rules considered. The same holds for the other macroeconomic variables, with the obvious exception of the nominal exchange rate. The domestic currency permanently depreciates under the Taylor and the smoothing rules.

A major difference with the flexible price benchmark concerns the responses of markups. With nominal rigidity, markups vary along the business cycle, staying above the steady state throughout most of the transition and they do so irrespectively of the monetary rule in place. The finding is a consequence of the price distortions that materialize with Calvo pricing. The possibility that a producer might not be able to adjust the price of its own variety in the future contributes to deter potential entrants, raising the real value of investments above entry costs. Massive exits, in turn, weaken competition among producers and reflect into higher margins of profit.

The responses in Figure 1 further show that deviations of inflation from target are negatively associated with the output gap, consistently with the view stressing the existence of a trade-off between inflation and output. Interestingly, the relation between these two variables depends on the exchange rate regime. With fixed exchange rates, the output gap is negative and high in the early stage of the transition, then it progressively reduces and eventually becomes positive towards the end of the transition. With floating regimes, instead, the output gap is negative during most of the transition, except for a positive spike on impact. Clearly, the inability to change the exchange rate makes it harder for monetary authorities to reproduce the macroeconomic dynamics that would
prevail with flexible prices, especially in the early stage of the transition. As adjustment proceeds, on the contrary, the distance with the natural exchange rate reduces and eventually disappears. Consequently, the pattern of the terms of trade under fixed exchange rates lags behind the one with floating regimes for most of the time.

In order to evaluate international spillovers, consider the responses of home aggregates to a rise in foreign entry costs as shown in Figure 2. The shock has a persistence of 0.9 as before.

The rise in the real price of foreign investments clearly discourages foreign entrants (not shown in Figure 2) for motives similar to those already discussed. The fall in the number of foreign producers, in turn, has two opposing effects on home aggregate demand. On the one side, it will depress world consumption via the rise in world interest rates (due to arbitrage between shares and bonds) and the variety effect (as long as less varieties of the Foreign good are available at a higher relative price, world consumption will reduce). As foreign producers gradually exit the market, on the other side, the relative price of each foreign variety increases, thereby appreciating the foreign terms of trade and shifting demand towards home goods. In the baseline calibration, the first effect prevails and both investments and output reduce at home.\footnote{With a high inter-temporal elasticity, however, output and investment spillovers turn positive on impact (not shown in figure).} As before, the fall in the number of producers is associated with positive markups throughout the transition.
Figure 3: Responses to 1 percent a rise in home productivity with Wicksellian policy (blue line), Taylor rule (green line), smoothing rule (black line) and fixed exchange rates (red line).

4.2.2 Productivity

Figure 3 shows the responses to a 1 percent rise in home productivity. The productivity shock has a persistence of 0.9 as in Bergin and Corsetti (2005).

Focus again on the scenario with optimal policy. The increase in productivity lowers marginal costs and real interest rates and has expansionary consequences for consumption and GDP. The favorable business environment, in turn, attracts more firms into the home market and the number of entrants increases. Note that the response of investments is very large (around 30 percent) as typical in RBC models without costs of capital adjustment.\textsuperscript{12} In my model, in fact, the real price of investment is constant absent exogenous changes in $f_E$ and it reduces to one unit of consumption absent such shocks, $\nu = 1$, exactly as in the standard RBC model. Importantly, however, this does not imply that the price of equity in units of the firm’s own output, $\nu \frac{P_H(h)}{P_H(h)}$, or nominal share prices are constant. Moreover, entry concentrates in the early stage of the transition with the number of entrants returning to the natural level in less than two years, while it translates into a gradual increase in the number of producers over time. As more producers compete in home markets, the price of home varieties gradually falls together with marginal costs at unchanged markups. The Wicksellian policy is counter-cyclical and the Home interest rate stays above the steady state throughout the transition. The hike in the interest rate is responsible for the appreciation of the Home currency and

\textsuperscript{12}In a closed economy context, Bilbiie-Ghironi and Méltiz (2007)) obtain a response of entrants to a productivity shock as high as 80 percent.
the Home terms of trade on impact. Over time, as the price of Home products gradually decreases and the interest rate returns to the steady state, the Home currency and the Home terms of trade depreciate before returning to the natural level.

Comparing the macroeconomic dynamics under nominal rigidity with the flexible price benchmark reveals a number of interesting features. The responses of output, the number of entrants and the stock of producers, although similar qualitatively, vary significantly in magnitude. This is a consequence of investments being twice as large in the Wicksellian scenario than under the alternative monetary rules. In this respect, the introduction of nominal rigidity appears to mitigate the excess volatility of investments that is typical of RBC models without costs of capital adjustment. The reason is easy to grasp: nominal rigidity translates into real rigidity through the real price of investment, which is the key driver of entry in the model. In order to see why, consider the dynamics of producer price inflation and markups. Under the optimal rule, the increase in productivity reduces marginal costs and the relative price of each product in the economy in exactly the same proportion, leaving markups unaltered at the firm level and in the aggregate as well. This implies that relative prices adjust completely, i.e. they move one to one with the productivity shock, and so does the real value of investments. Clearly, producer inflation is zero. With Calvo pricing, instead, relative prices within a given period will differ across firms depending on whether they can or cannot adjust the price of their own product, thereby introducing a real price distortion. The price distortion, in turn, deters potential investments, implying less than optimal responses in the number of entrants and the stock of production lines. Moreover, the disconnect between relative prices and marginal costs translates into variable margins of profits throughout the transition. Remarkably, the responses of markups in Figure 3 reproduce the typical counter-cyclical pattern that appears in the data.

Price distortions show up in the pattern of inflation. On impact, home producer prices fall driven by the decline in marginal costs. Over time, as the number of producers returns to the steady state, deflation gradually tends to zero. Notice that the responses of key macroeconomic variables depend on the exchange rate regime. With fixed exchange rates, the response of domestic producer prices to the rise in productivity is subdued. The gain in terms of price stability, however, comes at the cost of higher distortions in relative prices and therefore even a smaller incentive to invest. As entry dynamics weakens, in turn, markups stay closer to the steady state.

Interestingly, sticky prices appear to change the responses of nominal exchange rates relative to the flexible price benchmark along two dimensions. First, the sign of the response is completely reversed both on impact and over time: whenever the nominal exchange rate depreciates under the Taylor and the smoothing rule, it appreciates under the Wicksellian policy. Second, the Home currency displays a persistent appreciation with the Taylor and the smoothing rules, consistently with
ample evidence showing a significant degree of persistence in nominal exchange rate fluctuations. The finding reflects a different orientation of monetary policy. In a world with flexible prices, monetary policy simply accommodates changes in the natural interest rate, letting the exchange rate fluctuate consequently. This is possible only at the cost of higher exchange rate variability when prices are sticky. I stress that excess volatility materializes even with a moderate aggressiveness towards inflation as in the rules considered. As it will be apparent soon, the type of monetary rule in place has a major role in transmitting shocks worldwide.

Figure 4 reports the responses of domestic macroeconomic aggregates to a 1 percent rise in foreign productivity. The persistence of the shock is 0.9 as in the Home country.

The boost in productivity has a positive impact on world consumption in all the regimes considered, independently on whether it originates at home or abroad. In floating regimes, the responses of consumption to foreign productivity are very close indeed to the responses in Figure 3. The motives are essentially the same: the rise in productivity lowers real interest rates and attracts investments (abroad, in this case), thereby stimulating world consumption.\textsuperscript{13} The rise in foreign productivity propagates its effects worldwide through the movements in the terms of trade. In the flexible price scenario, the foreign terms of trade (and the foreign currency) deteriorate during most of the transition, except for an appreciation on impact, shifting demand towards foreign goods. In the Home country, in turn, improving terms of trade and higher prices for domestic varieties increase the real

\textsuperscript{13}The response of consumption with fixed exchange rates is similar to the one in Figure 3 yet it is much smaller in the early periods and larger towards the end of the transition.
price of investments and induce firms to exit Home markets. As already stressed, the response of investments is very large (-20 percent) and translates into a substantial fall in the number of home producers. Consequently, Home GDP drops in the early stage of the transition despite the expansion in world consumption.

Comparing responses across monetary rules reveals a number of interesting features in international output and investment spillovers. First, the response of investments under all three rules considered is much weaker than with flexible prices (less than 1 percent relative to a fall as high as 20 percent with flexible prices). The reason lies in the price distortions due to Calvo pricing which, as already discussed, act as sand in the wheels of the entry mechanism. Second, the response of investments to the boost in foreign productivity may even turn positive, as it appears to be the case with fixed exchange rates. The finding is a consequence of the fall in the real price of Home investments which occurs in the early stage of the transition when Home prices are insulated from the inflationary effects of a depreciating Home currency (as a matter of fact, Home producer inflation is negative in the first year of the transition). The combination of declining prices and a higher number of producers under fixed exchange rates, in turn, leads markups below their natural level throughout the transition. Finally, output spillovers are positive both on impact and along the transition, driven by the improvement in the home terms of trade (note that the improvement is larger with flexible exchange rates). I stress that output adjustment takes place mostly along the intensive margin, with an increase in firms’ size (not shown in Figure 4).

4.2.3 Monetary policy

Figures 5 and 7 show the responses to a one percent purely transitory fall in, respectively, domestic and foreign interest rates for different values of the inter-temporal elasticity. The black line corresponds to the case of low elasticity, $\rho = 0.16$, and the blue line to $\rho = 1.16$. The responses are calculated under a symmetric smoothing rule.

The responses of consumption in Figures 5 and 7 are identical as one would expect with perfect risk sharing. The monetary expansion, wherever it is originated, boosts world demand as long as prices are sticky, leading to a spike in world consumption on impact and throughout the first year of the transition. Over time, as relative prices slowly return to their natural levels, world consumption declines for a while before converging to the steady state. Clearly, the responses of consumption are much more volatile when the inter-temporal elasticity is small (on impact, they are approximately ten times larger with a low $\rho$). Notice that despite the shock is purely transitory, the dynamics of consumption and other macroeconomic aggregates display a significant degree of persistence as one
observes in the data. In my setup with endogenous entry, the finding holds, albeit over a shorter time horizon, also with monetary rules that do not feature interest rate smoothing (Figures 8 and 9).

The rise in global demand must be accommodated by a corresponding increase in global output. Comparing the responses in Figures 5 and 7, it appears that the origin of the shock is crucial for understanding whether adjustment takes place mostly at home or abroad and along the intensive or the extensive margin. To start with, focus on the home monetary shock. The monetary easing fuels domestic inflation (Figure 6), thereby contributing to devalue the domestic currency, deteriorate the home terms of trade and reduce the relative price of domestic varieties, $P_h/P_H$. As a consequence, the real value of investments stays above the steady state throughout the transition and discourages potential entrants despite the favorable demand conditions. Investors prefer to accommodate the rise in demand by increasing the scale of production, i.e. output per firm lies above the steady state for most of the transition. In so doing, in fact, they can take advantage of the fall in real wages that is associated with declining terms of trade and maintain markups well above the steady state. I stress that this holds independently on the value of the inter-temporal elasticity. The fall in the number of entrants, in turn, is responsible for the drop in domestic output in the early stage of the transition. Over time, as world demand cools down and exit weakens, output will gradually return towards the natural level. The finding that investments in new varieties might be negatively associated with a monetary easing accords with the results in Bilbiie, Ghironi and Mélitz (2007). In their model, exit is driven by the rise in the real value of investments that materializes with a monetary easing (a drop in the interest rate directly reduces the real return on bonds and, through arbitrage, also the return on shares. This in turn requires an increase in share prices and therefore in entry costs). In my open-economy setup, the real value of investment might either increase or fall depending on the country of origin of the shock. As it will be apparent soon, a foreign monetary easing may well lead to an increase in the number of domestic entrants.

Now, consider the responses of domestic variables to a monetary easing abroad (Figure 7). In the early stage of the transition, the burst in foreign demand is associated with an appreciation of the home currency and an improvement in the home terms of trade, both $T$ and $\varepsilon$ fall. World expenditure therefore switches in favour of foreign goods. Nonetheless, home GDP rises on impact and remains above the steady state for about a year, precisely 3 quarters with low $\rho$ and 5 quarters with high $\rho$, driven by the hike in world consumption. Over time, as consumption declines output stays below the natural level for most of the transition.

I stress that changes in GDP might take place mostly along the intensive or the extensive margin depending on the inter-temporal elasticity. Potential investors in home markets, in fact, face two
Figure 5: Responses to a transitory fall in the home interest rate with low (black line) and high elasticity (blue line).

Figure 6: Responses to a transitory fall in the home interest rate (continued) with low (black line) and high elasticity (blue line).
Figure 7: Responses to a transitory fall in the foreign interest rate with low (black line) and high elasticity (blue line).

opposing forces. On the one side, they are induced to invest more in the very early stage of the transition so as to profit from the favorable demand conditions that materialize in the aftermath of the monetary easing. On the other side, however, the attractiveness of the business environment may deteriorate as a consequence of a temporary hike in marginal costs. In the first periods after the shock, in fact, real wages increase (not shown in Figure 7), driven by the improvement in the home terms of trade. The first effect prevails when the inter-temporal elasticity is high, implying a positive correlation between investments in new varieties and monetary innovations.

Figures 8 and 9 display the responses to transitory interest rate shocks in the Home and the Foreign country in the benchmark calibration under different specifications of the interest rate rule followed by the monetary authorities in both countries: the black line refers to the Taylor rule with a producer inflation target, the green line to the Taylor rule with a consumer inflation target and the blue line to the fixed exchange rate regime.

The responses in Figures 8 and 9 are qualitatively identical to those in Figures 5 and 7, except for the regime with fixed exchange rates. The major difference between the smoothing and the Taylor rules concerns the higher persistence of macroeconomic aggregates under the former rule.

Focusing on Figure 8, I stress that targeting producer or consumer prices does not affect the responses to domestic interest rate shocks in a significant way. The responses to a drop in the foreign interest rate (Figure 9), on the contrary, are much more pronounced under the target on producer inflation. The reason is that stabilizing consumer inflation requires Home monetary authorities to offset the inflationary pressure from abroad by increasing the home interest rate. This in turn tends
Figure 8: Responses to a transitory fall in the home interest rate with producer inflation target (black line), consumer inflation target (green line) and fixed exchange rates (blue line)

to stabilize the nominal exchange rate and therefore reduce international spillovers. In Figure 9, the responses under the target on consumer inflation are very close indeed to those with fixed exchange rates. Not surprisingly, fixing the exchange rate completely insulates the economy from monetary shocks abroad.

5 Conclusions

This paper has analyzed the role of producer entry for global monetary policy and the propagation of the international business cycle in a two-country general equilibrium (DSGE) model with monopoly distortions and imperfect price adjustment. By introducing endogenous producer entry into the model of Benigno and Benigno (2008), it has provided a theory for the joint determination of exchange rates and product varieties under interest rate rules. I have considered two floating regimes characterized by Taylor rules with and without interest smoothing and a regime with fixed exchange rates and compared the macroeconomic dynamics under these rules with its flexible price counterpart. Numerical examples have documented the responses of key endogenous variables to country-specific shocks to aggregate productivity, entry costs and interest rates.

The paper shows that endogenous product variety affects different dimensions of the international cyclical transmission. First, firms’ entry introduces an endogenous persistence into the patterns of macroeconomic aggregates, producer prices, exchange rates and the terms of the trade consistently with stylized facts. Exchange rates, in particular, are found to be non-stationary under floating
Second, I show that entry reacts to interest rate innovations as long as prices are fixed, extending to open economies the monetary transmission mechanism in Bilbiie, Ghironi and Mélitz (2007) and Bergin and Corsetti (2005). Differently from these contributions where interest rate changes are, respectively, either positively or negatively correlated with entry, I stress that an easing of the global monetary stance might in principle attract or deter investments along the extensive margin, i.e. investments in new varieties. In my setup, interest rate setting has two opposing effects on entry. On the one side, a drop in the interest rate, by inducing more consumption today, reduces the resources available for investment and therefore discourages potential entrants. The boost in demand, on the other side, creates a more favorable business environment thereby attracting new investors. Numerical simulations show that the first effect prevails whenever the domestic currency depreciates, as when the monetary easing originates at home. With a depreciating currency, in fact, domestic producers are able to raise the relative price of their own variety yet limiting the loss in terms of their market shares. The opposite may occur under a foreign monetary expansion.

Finally, numerical examples show that the major differences in the cyclical properties of endogenous variables with sticky prices and their flexible price counterpart concern the responses of markups and the magnitude of the entry effect. I stress that nominal rigidity acts as sand in the wheels of the entry mechanism by introducing persistent real price distortions across firms. The disconnect between relative prices and marginal cost, on the other side, translates into counter-cyclical margins of profits over the cycle.
References


